Methods for Finding Globally Maximum-efficiency Impedance Matching Networks with Lossy Passives

Chandrakanth R Chappidi and Kaushik Sengupta,
Electrical Engineering,
Princeton University

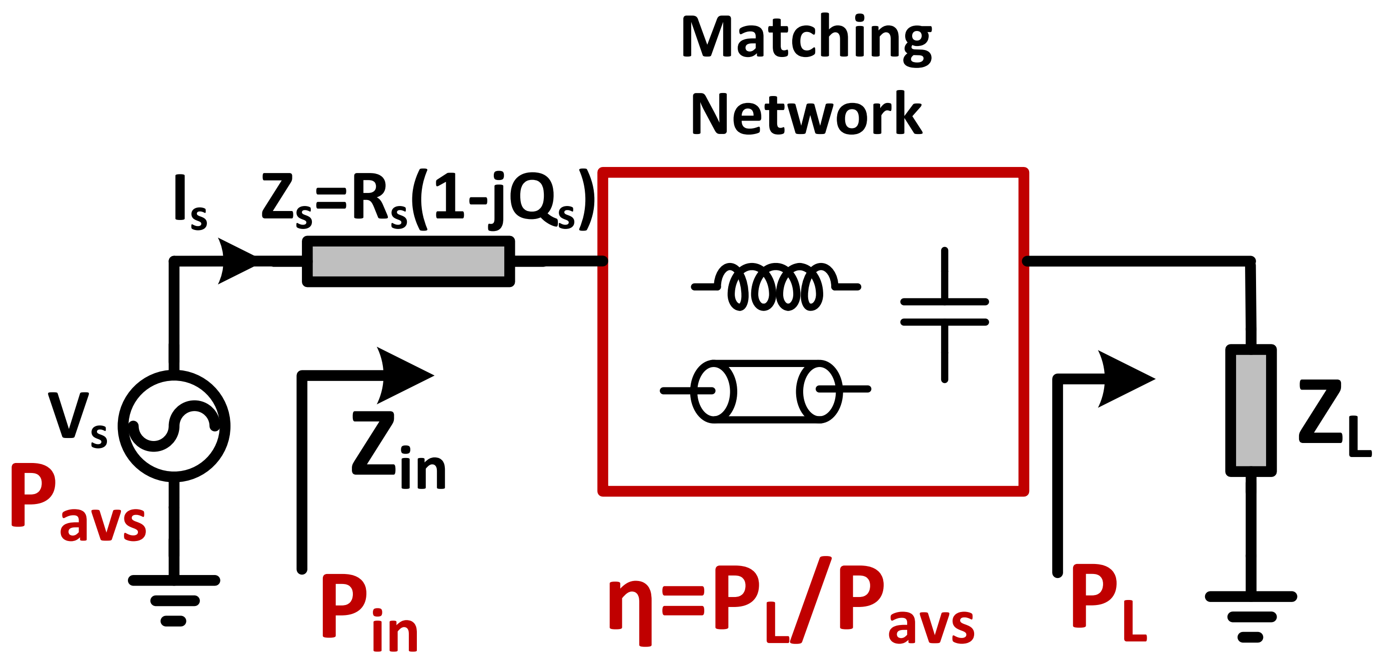
Outline

- Concept of Impedance Matching and Motivation
- Non Optimality of Conjugate Matching Networks
- Globally Optimal Impedance Matching Networks
- Design Examples

Outline

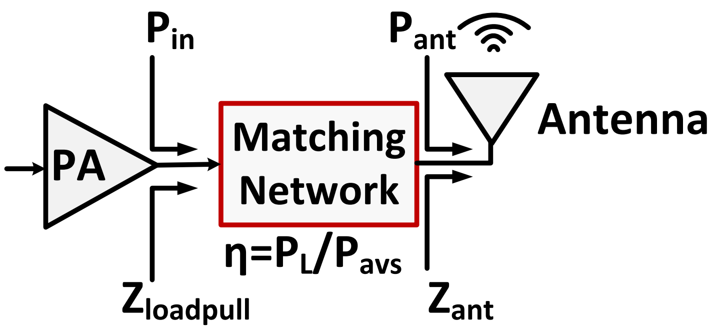
- Concept of Impedance Matching and Motivation
- Non Optimality of Conjugate Matching Networks
- Globally Optimal Impedance Matching Networks
- Design Examples

Impedance Matching



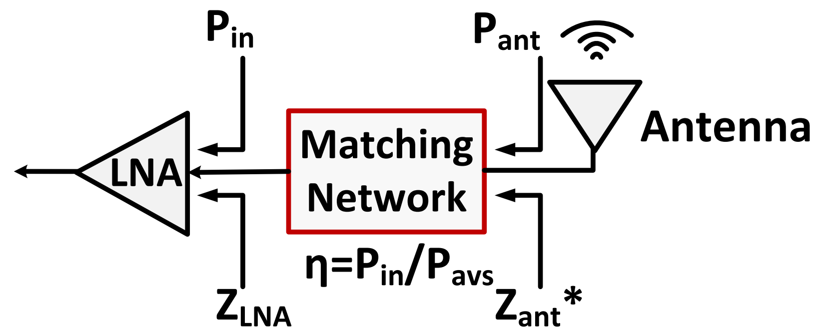
- For maximum power transfer from the source, $Z_{in,opt} = Z_s^*$
- **Matching network transforms Z_L to $Z_{in,opt}$ at the given frequency of operation.**

Impedance Matching Examples



Power Amplifier

$\eta_{total}=\eta_{PA}*\eta_{MN}$

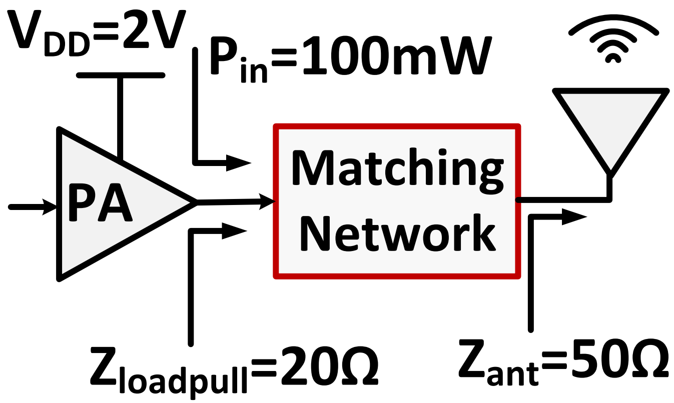


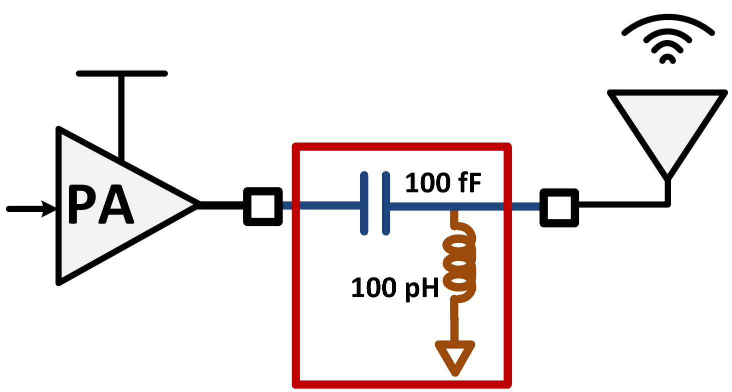
Low-noise Amplifier

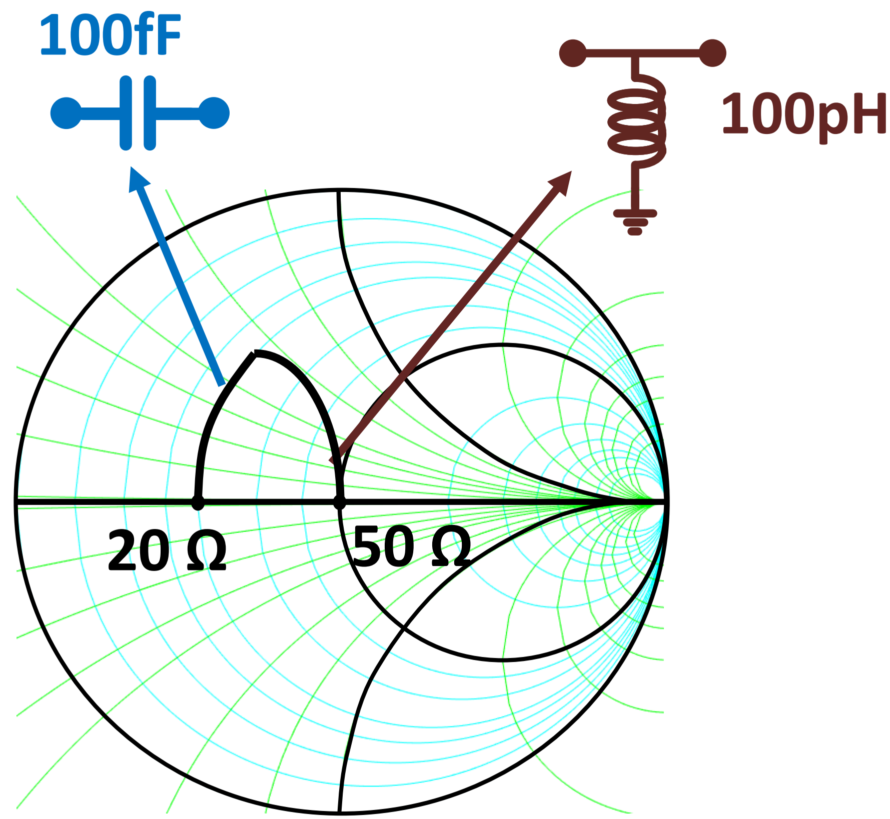
$NF \sim NF_{LNA}+\eta_{MN}(dB)$

- Maximize power transfer to the antenna at the transmitter front-end.
- **Loss in matching network directly affects PA/Tx efficiency.**
- Maximize power transfer to the LNA at the receiver front end
- **Loss in matching network directly affects LNA/Rx noise figure.**

Impedance Matching Example on Smith Chart



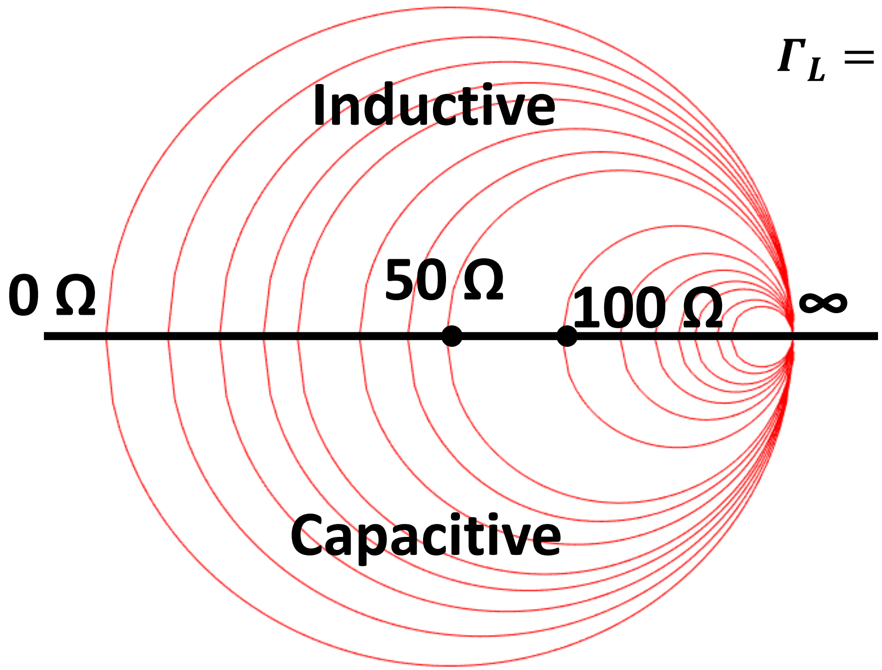




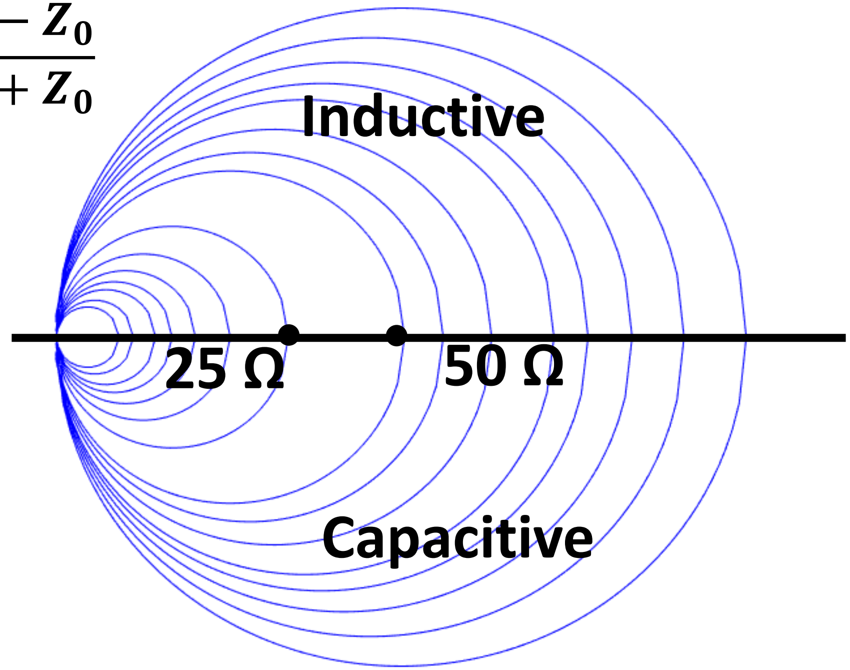
- Impedance transformation can be represented by a path on the Smith Chart

Smith Chart as a Graphical Representation

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$



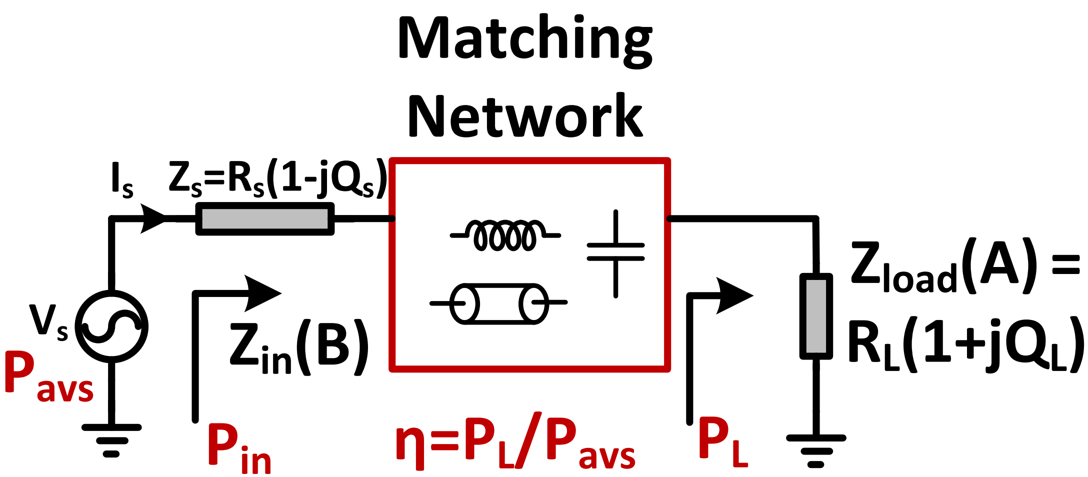
Constant Resistance circles

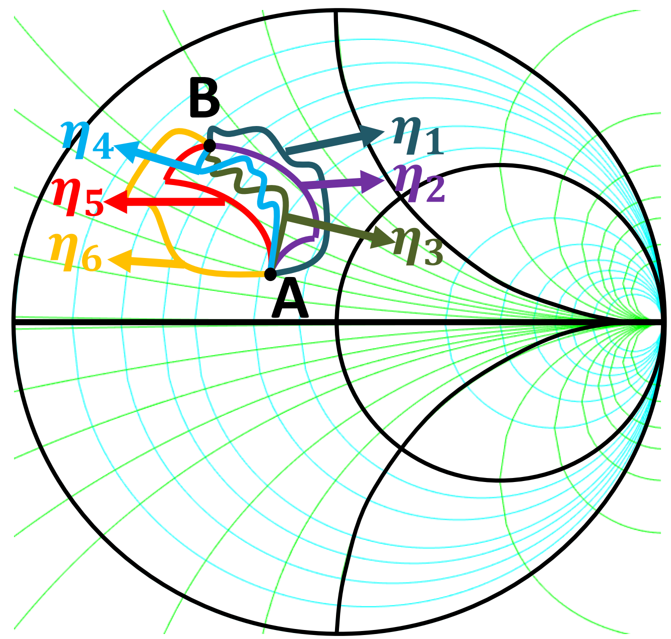


Constant Conductance circles

- Smith Chart is a graphical representation of impedance and transformation between impedances

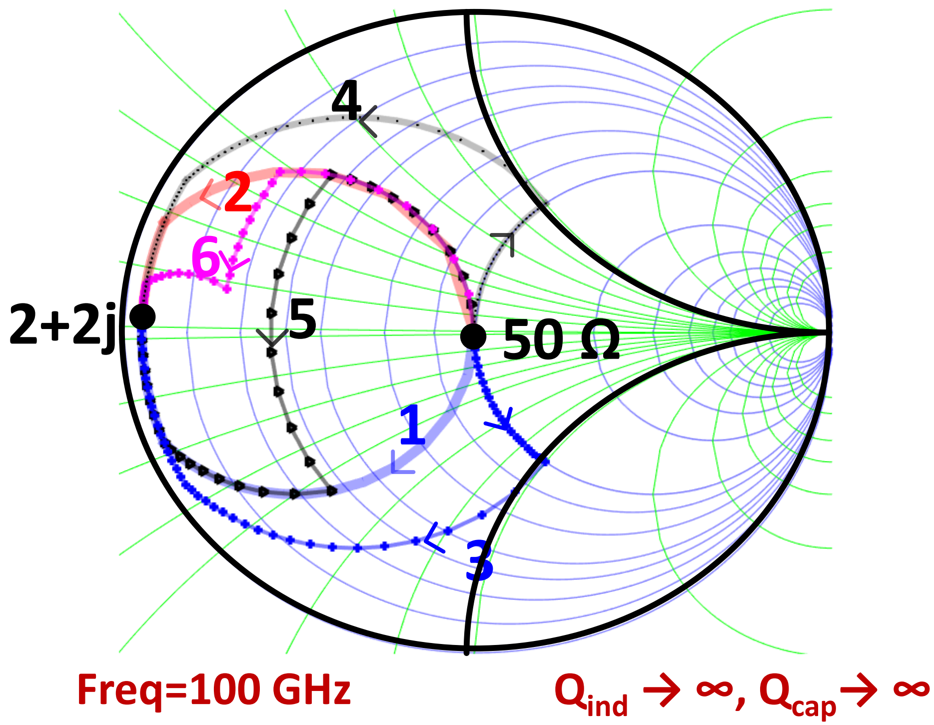
Number of Possible Paths in Transformation

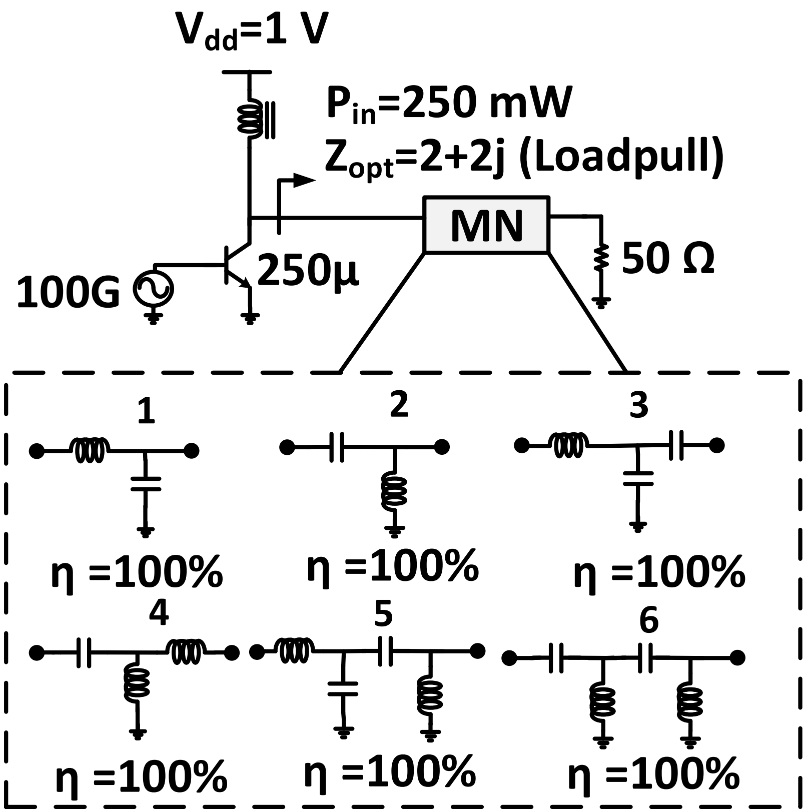




- The matching network for given Z_L and Z_{in} is not unique
- **Ideally, there are infinite possible paths between the impedances (A and B) on the Smith Chart: Which one's the most efficient?**

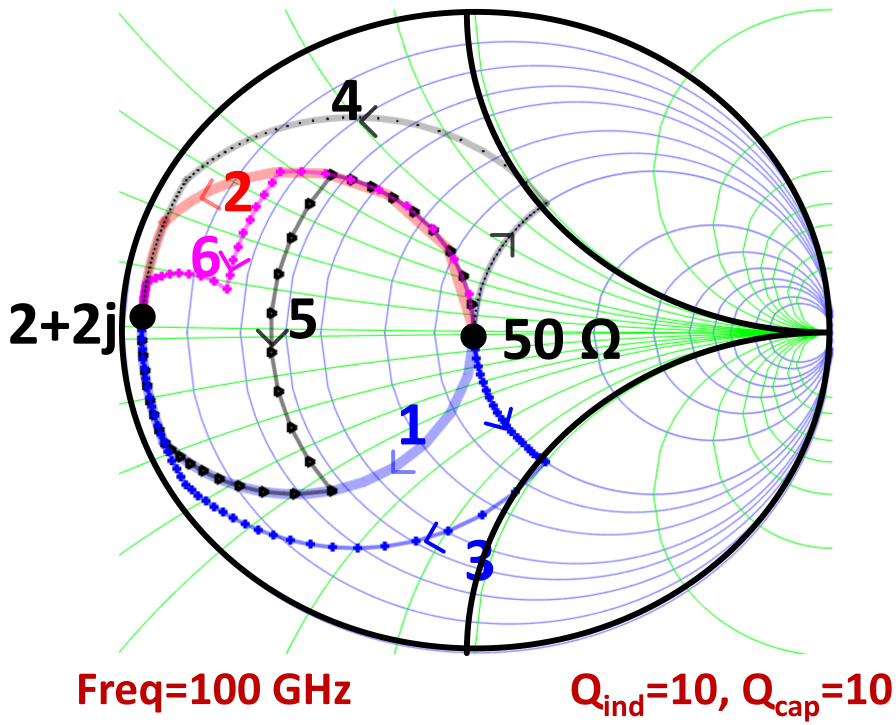
Matching Network with Lossless Passives

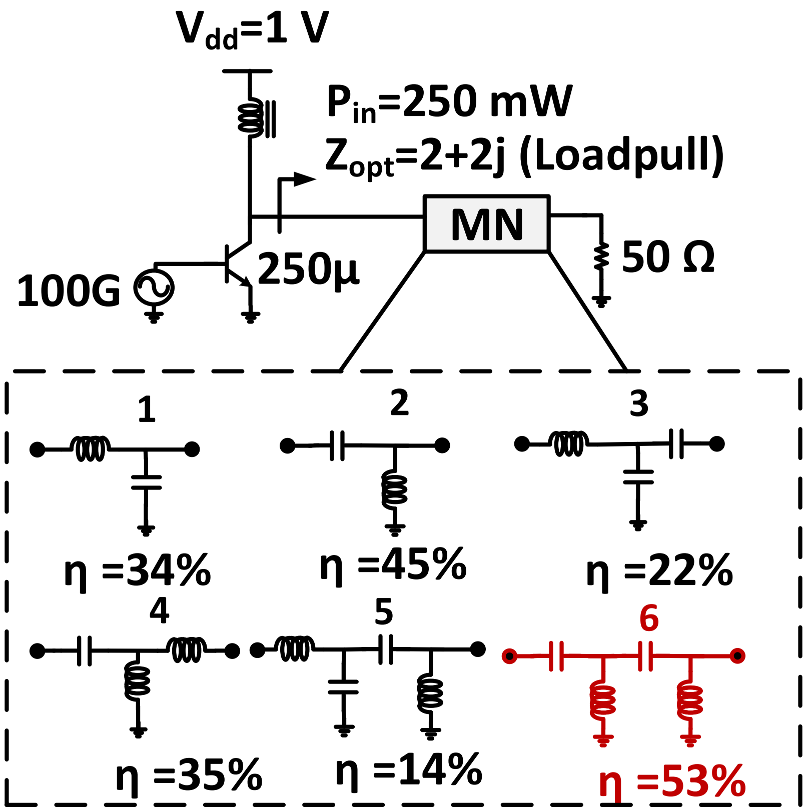




- Of course, for lossless passives, the efficiency of all paths are the same (100%)
- However, they will have different bandwidths.

Matching Network with Lossy Passives



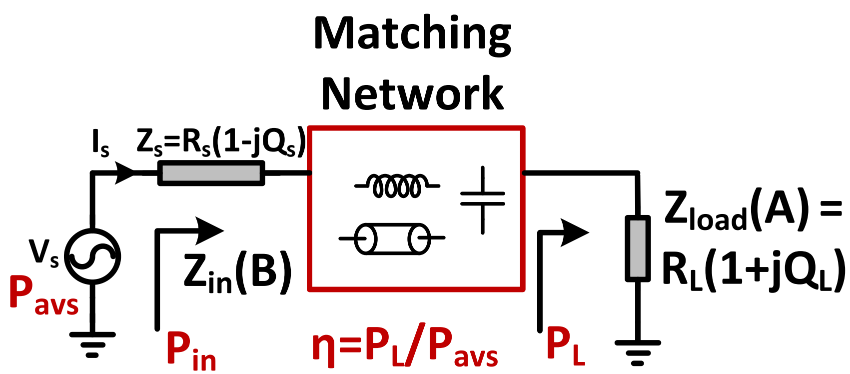


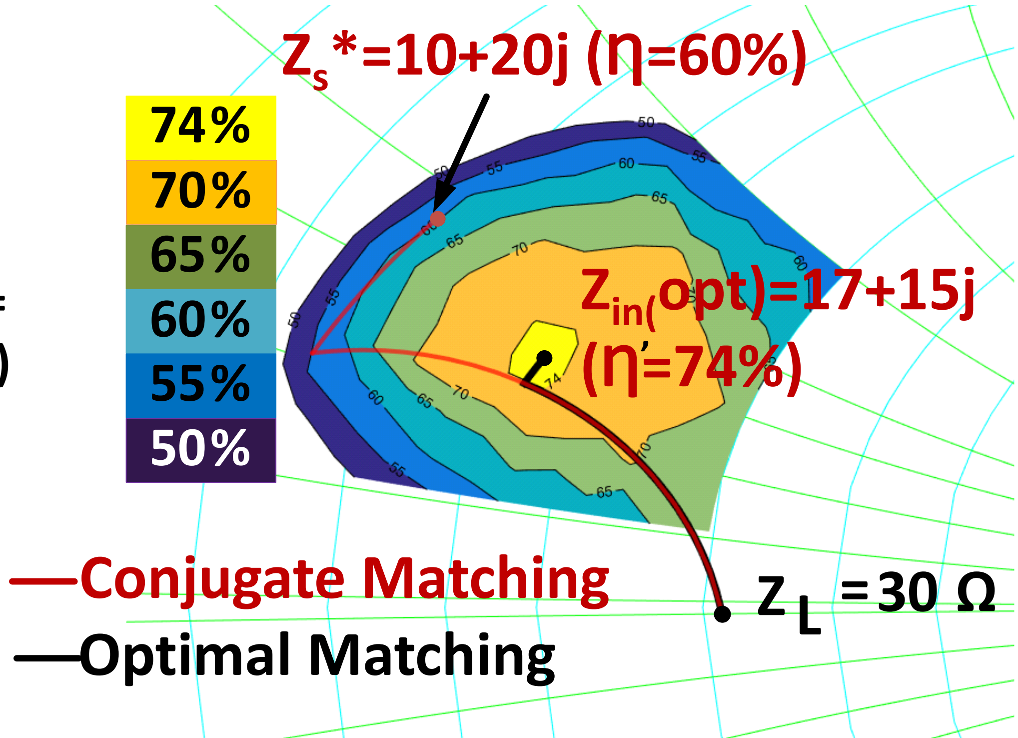
- **Efficiency of each path (representing different matching network) is different for lossy passives.**
- η_{max} (observed here) = 53% for a two-stage network.

Outline

- Concept of Impedance Matching and Motivation
- Non Optimality of Conjugate Matching Networks
- Globally Optimal Impedance Matching Networks
- Design Examples

Non-optimality of Conjugate Matching

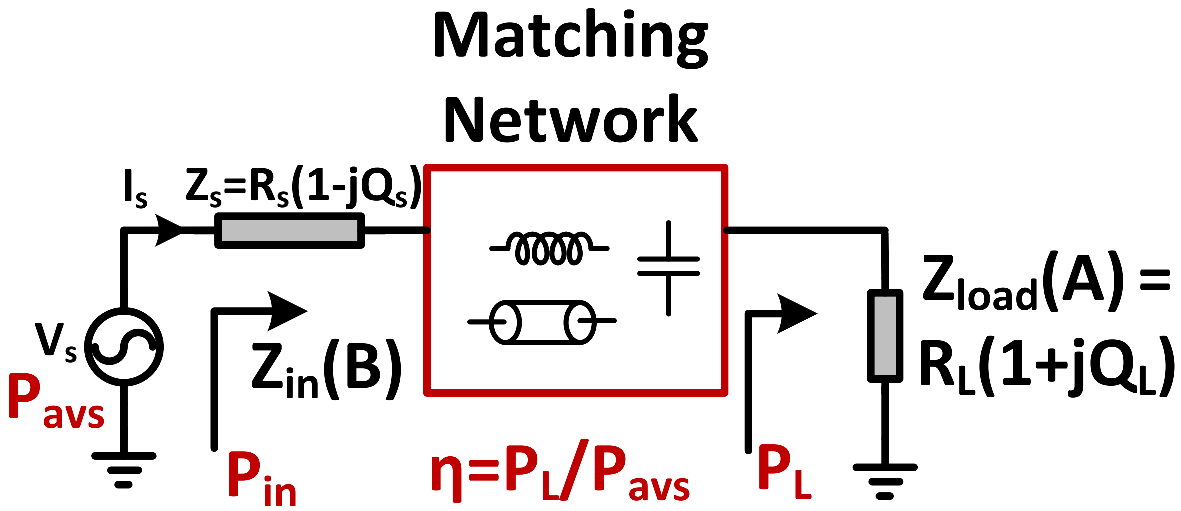




- $Z_s=R_s(1+Q_s)$=10-20j, Q_{ind}=5 , Q_{cap}=10

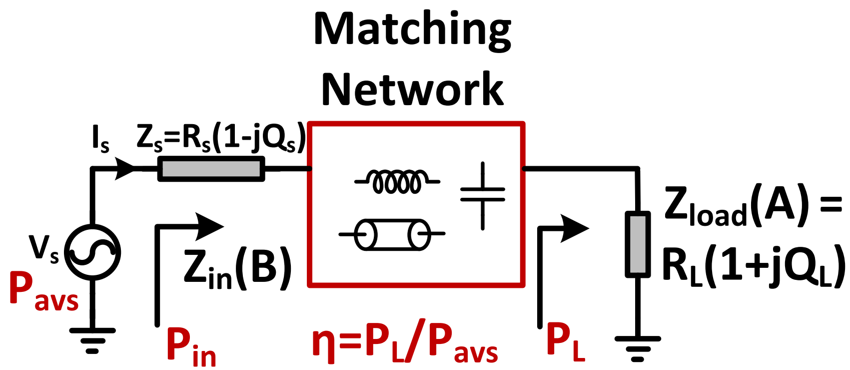
- Conjugate matching may be sub-optimal because more power may be wasted in transforming to Zin=Zs*
- **Maximizing P_{in} does not necessarily maximize P_L**

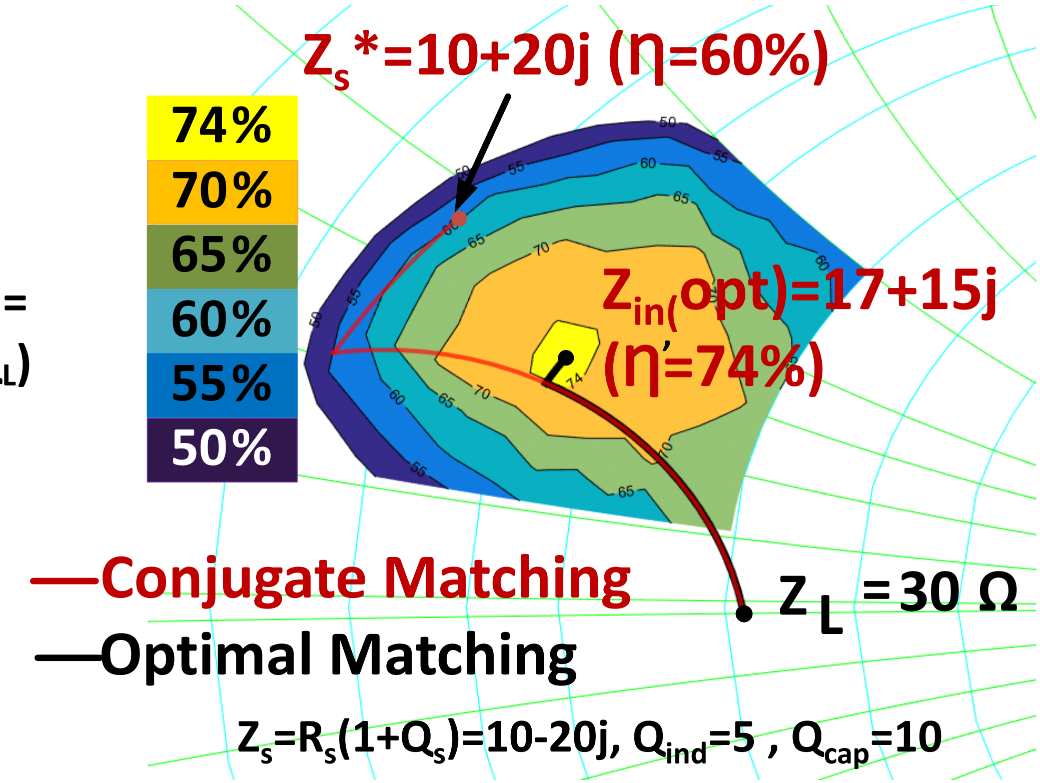
Analysis of Optimal Inductor-only matching



- $P_{reac} = \frac{1}{2} I_s^2 X_{in} = P_{ind} - P_{cap} + P_{reac,L}$
- $P_{loss} = \frac{P_{ind}}{Q_{ind}} + \frac{P_{cap}}{Q_{cap}} \geq \left| \frac{P_{ind} - P_{cap}}{Q_{ind}} \right| = \frac{P_X - P_{reac,L}}{Q_{ind}}$
- $X_{in,opt} = R_s \frac{Q_{ind}^2 Q_s - Q_s - 2Q_{ind}}{1+Q_{ind}^2}$, $R_{in,opt} = R_s \frac{2Q_{ind}Q_s + Q_{ind}^2 - 1}{1+Q_{ind}^2}$, $P_{Load,Max} = P_{avs} \frac{1+Q_{ind}^2}{Q_{ind}(Q_{ind}+Q_s)\left(1-\frac{Q_L}{Q_{ind}}\right)}$

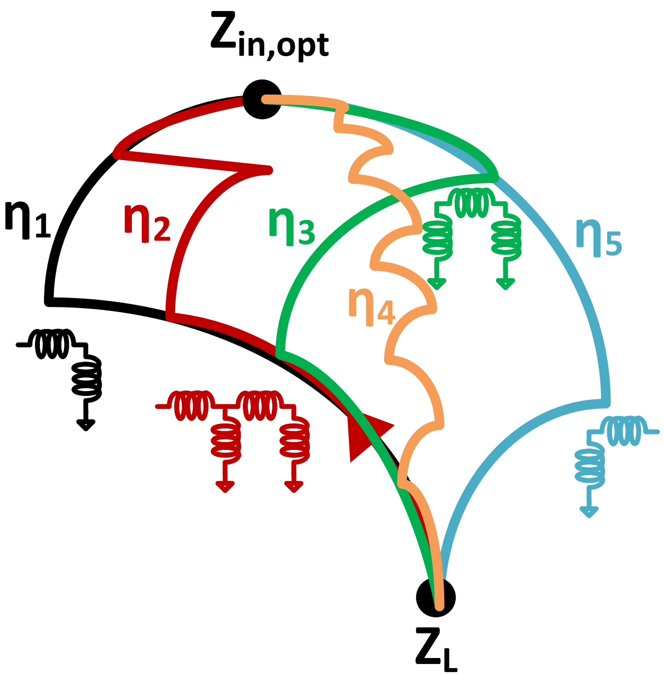
Analysis of Optimal Inductor-only matching



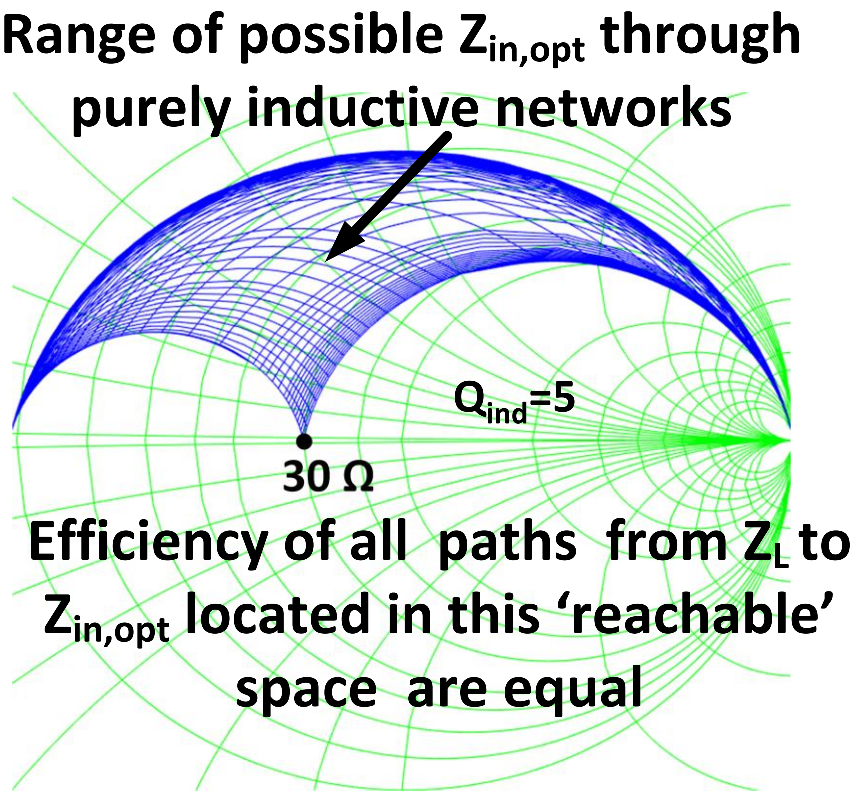


- $R_{in,opt} = R_s \frac{2Q_{ind}Q_s + Q_{ind}^2 - 1}{1+Q_{ind}^2} = 17, \; X_{in,opt} = R_s \frac{Q_{ind}^2 Q_s - Q_s - 2Q_{ind}}{1+Q_{ind}^2} = 15,$
- $P_{Load,Max} = P_{avs} \frac{1+Q_{ind}^2}{Q_{ind}(Q_{ind}+Q_s)\left(1-\frac{Q_L}{Q_{ind}}\right)}$ =0.74 , $P_{in} = 0.9P_{avs}$

Inductor-only matching (Smith Chart Coverage)



$\eta_1 = \eta_2 = \eta_3 = \eta_4 = \eta_5$

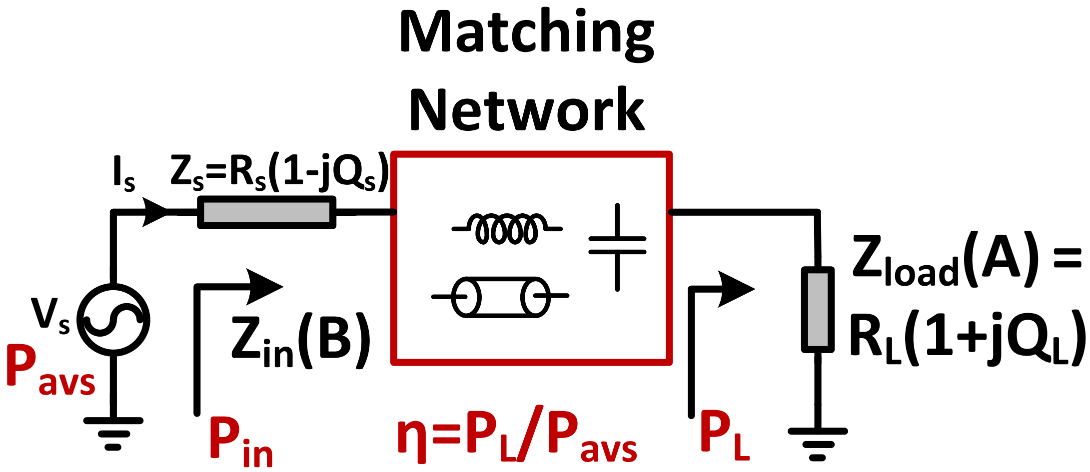


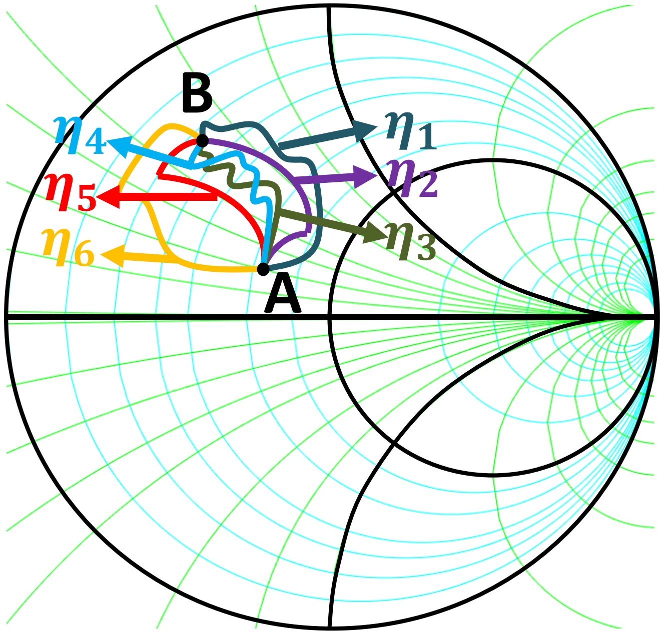
- Inductor-only paths are optimal and have the same efficiency
- **Given Z_L, there is a limited coverage on the Smith Chart for Inductor-only matching**

Outline

- Concept of Impedance Matching and Motivation
- Non Optimality of Conjugate Matching Networks
- Globally Optimal Impedance Matching Networks
- Design Examples

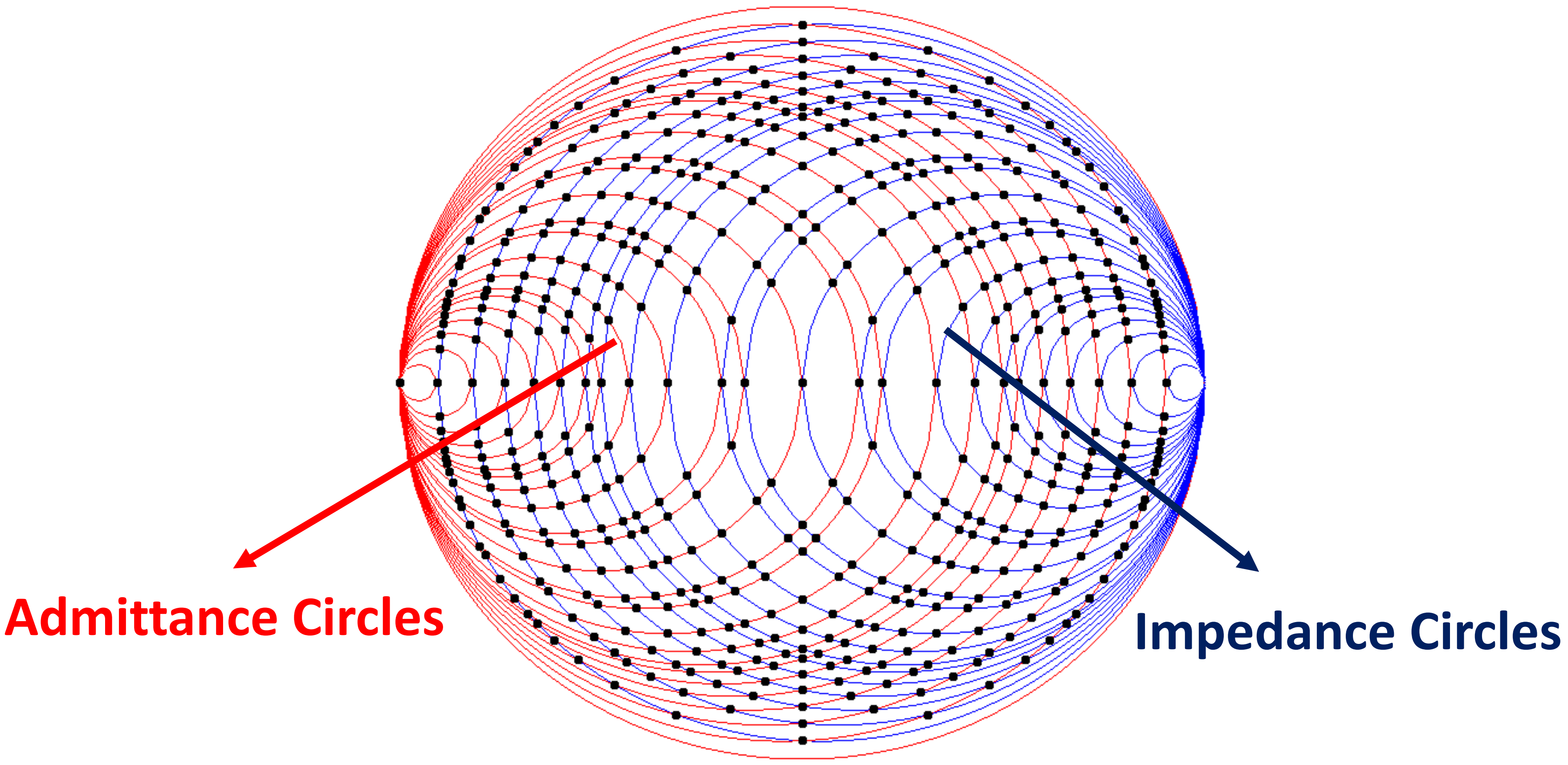
Globally Optimal Path between Z_A and Z_B





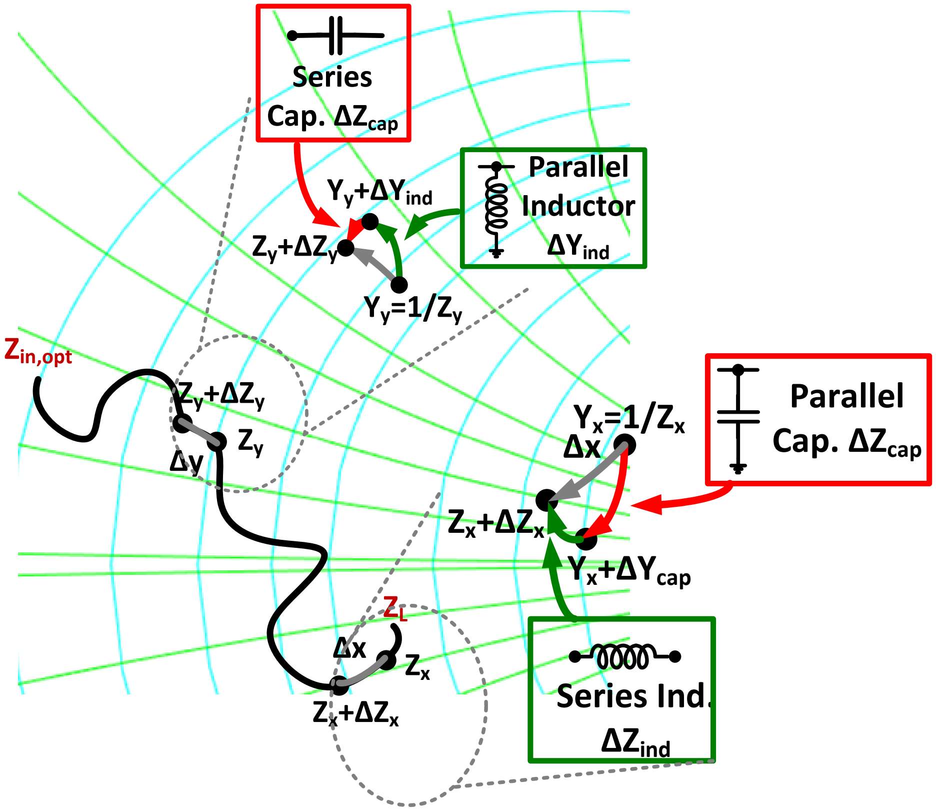
- **What is the globally optimally efficient path between two impedances Z_A and Z_B?**
- Any arbitrary path on the Smith can be realized with infinitesimally small/large elements, if necessary.

Globally Optimal Path between Z_A and Z_B

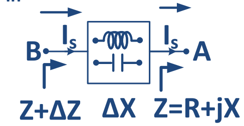


- **Discretize the Smith Chart Space into infinitesimally close admittance and impedances.**

Globally Optimal Path between Z_A and Z_B



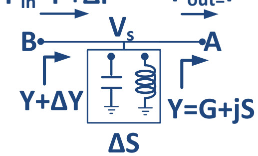
Loss in series element



ΔZ= jΔX+ |ΔX|/Q

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\frac{I_s^2}{2}R}{\frac{I_s^2}{2}\left(R + \frac{|\Delta X|}{Q}\right)} = \frac{R}{R + \frac{|\Delta X|}{Q}} = \frac{1}{1 + \frac{|\Delta X|}{RQ}}$$

Loss in parallel element



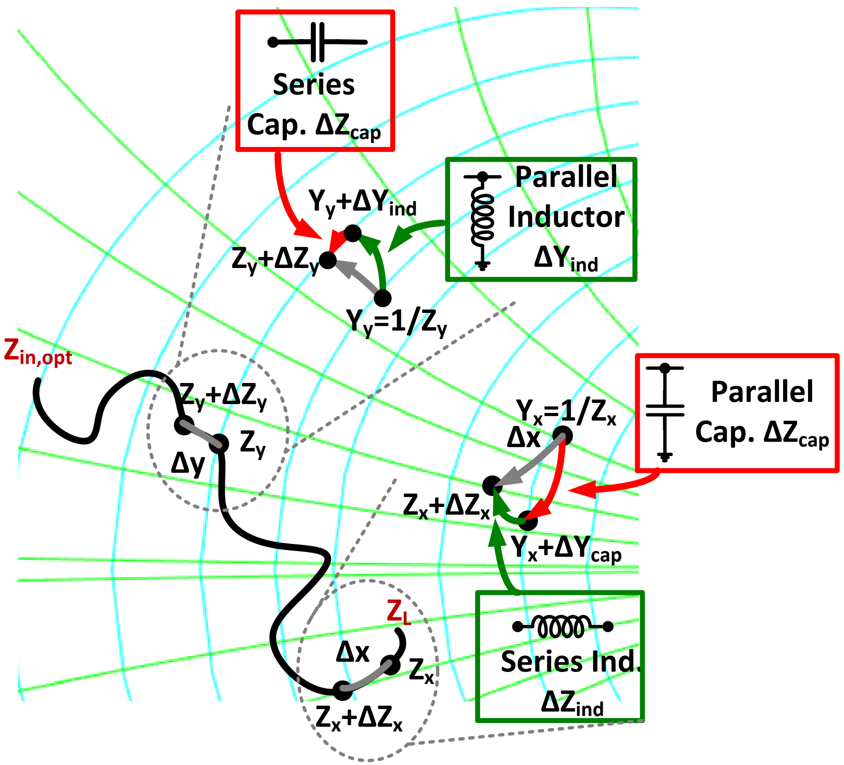
ΔY= jΔS+ |ΔS|/Q

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\frac{V_s^2}{2}G}{\frac{V_s^2}{2}\left(G + \frac{|\Delta S|}{Q}\right)} = \frac{G}{G + \frac{|\Delta S|}{Q}} = \frac{1}{1 + \frac{|\Delta S|}{GQ}}$$

$$P_{loss} = \int dP \approx \sum \Delta P \qquad \eta_{path} = \prod_{i=1}^{n} \eta_i$$

- **Efficiency of any arbitrary path can be reduced to the product of efficiencies of the constituent sub-paths.**

Globally Optimal Path between Z_A and Z_B



$$P_{loss} = \int dP \approx \sum \Delta P$$

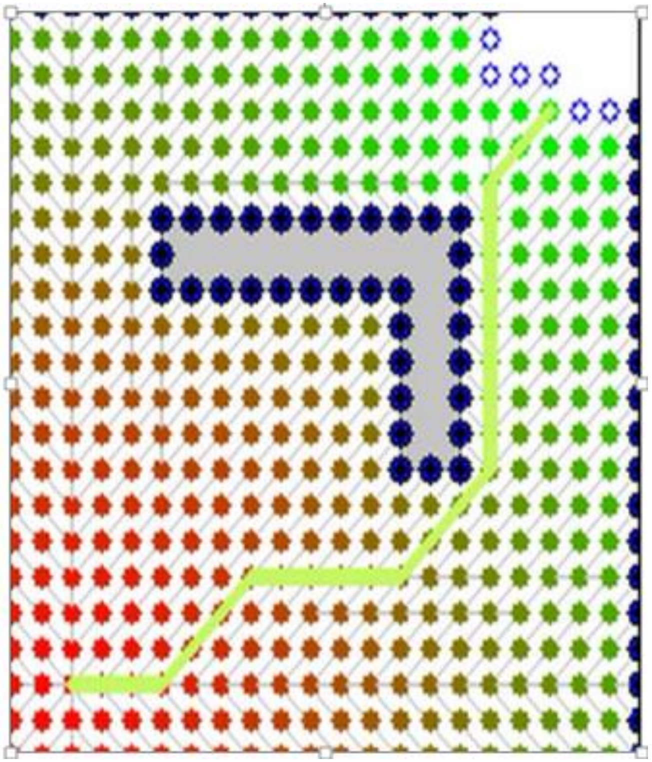
$$\eta_{path} = \prod_{i=1}^{n} \eta_i$$

Defining $d_i = -\ln(\eta_i) \Rightarrow d_{tot} = \sum d_i$

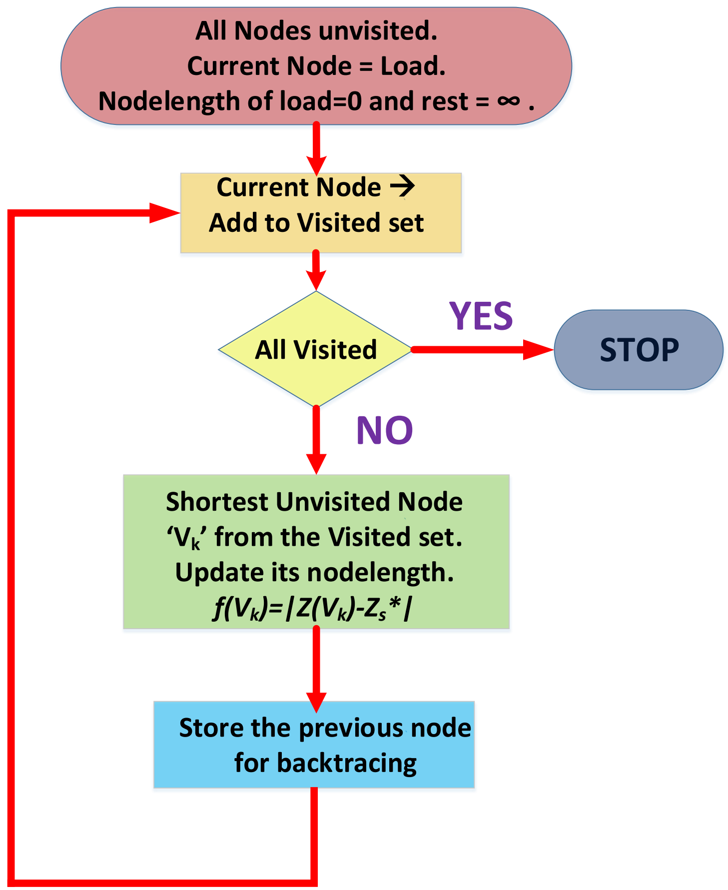
Maximize $\eta \Rightarrow Minimize\ d_{tot}$

- **Problem of optimally efficient matching network reduces to finding the shortest distance in a transformed space.**

Shortest Path (Maximally Efficient) Algorithm: Djikstra's Algorithm

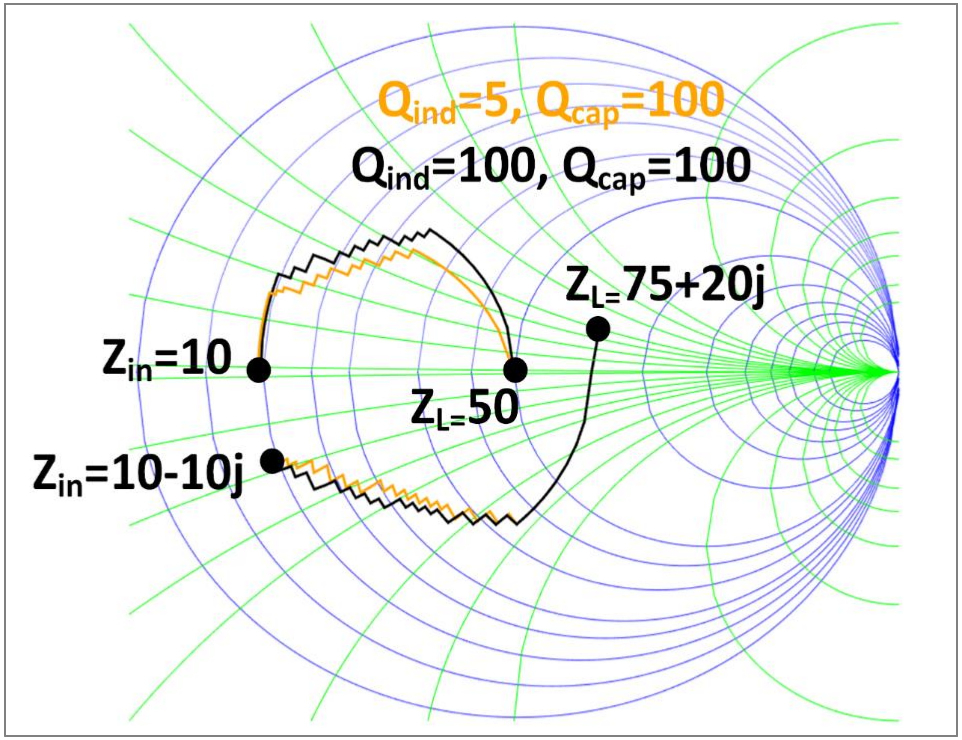


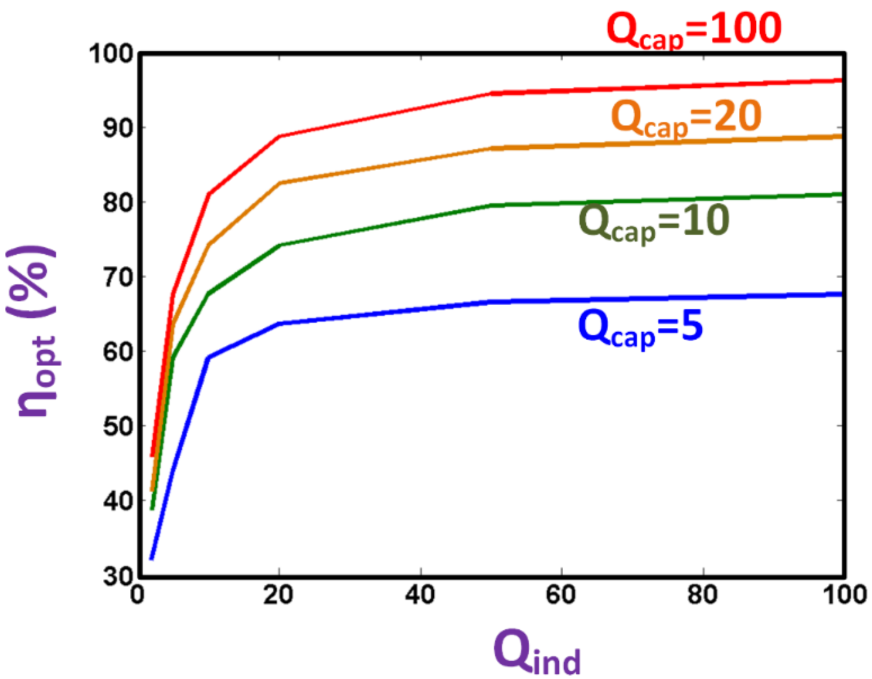
Djikstra's Algorithm (Wikipedia)



- Globally maximally efficient algorithm can be obtained from Djikstra's Algorithm with complexity $O(V^2)$(V=no. of nodes in the graph)

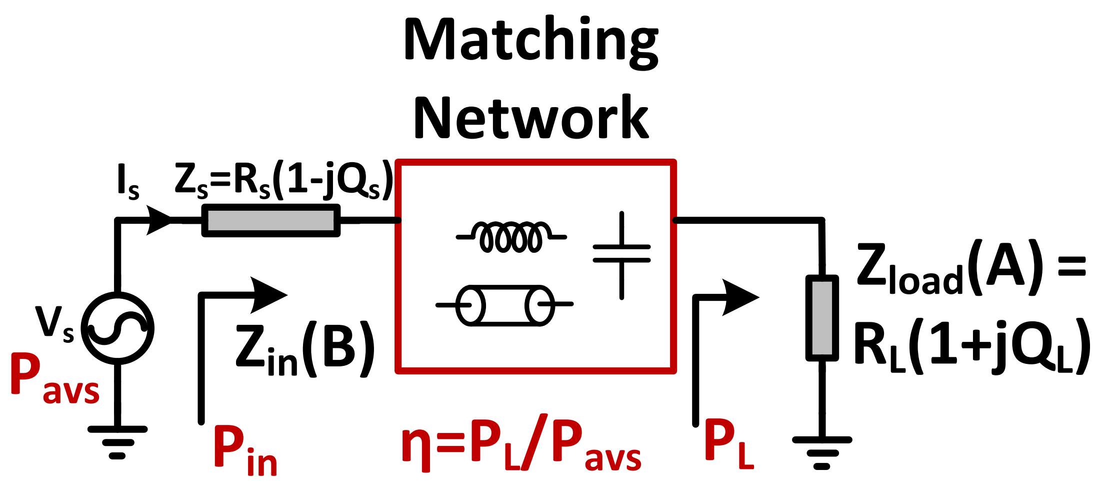
Globally Optimally Efficient Path (Example)





- Optimal paths vary with passive quality factors.
- Globally optimal paths can employ elements with infinitesimally small/large values.

Globally Optimally Efficient Power Transfer

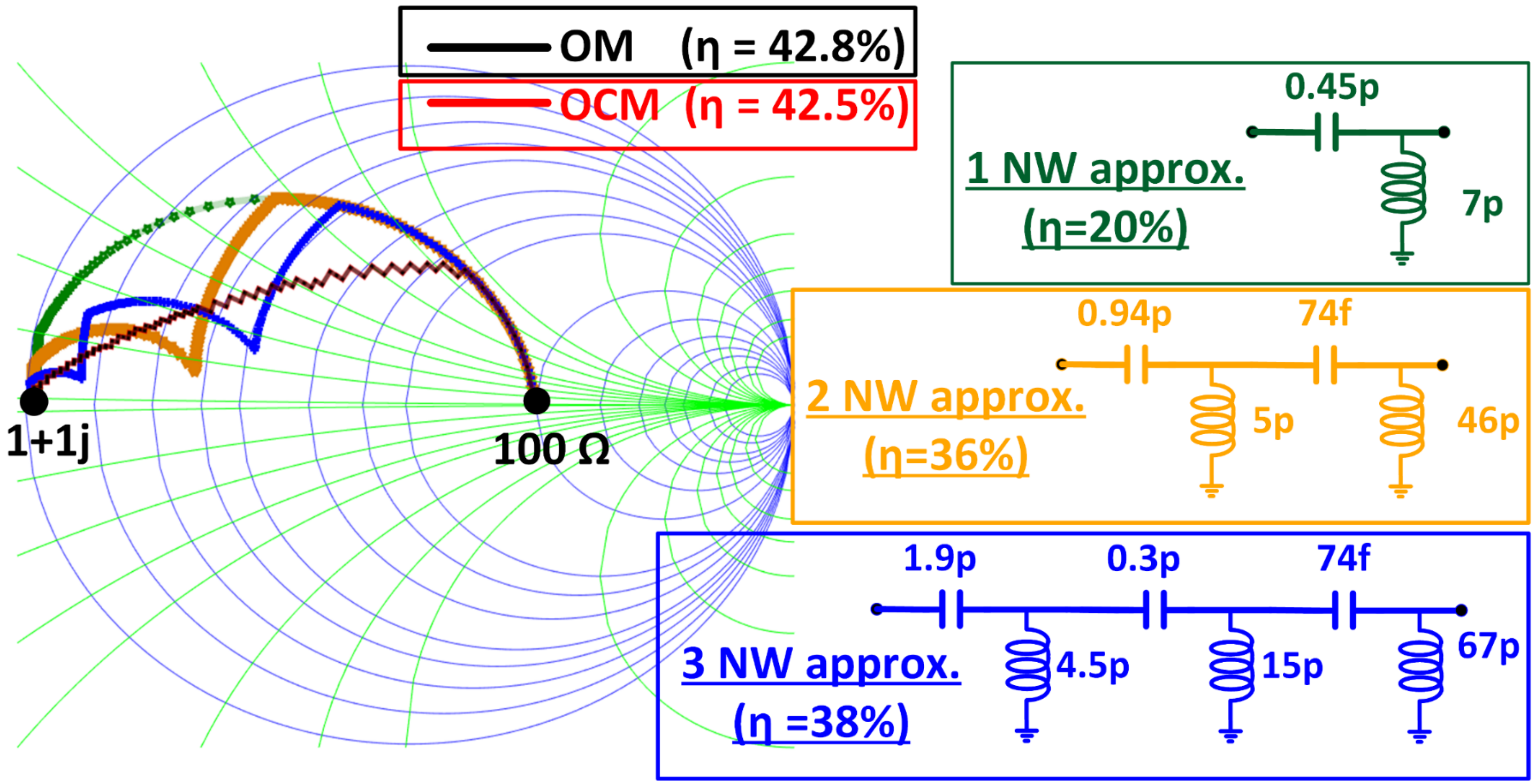


- $\eta_s = \frac{P_{in}}{P_{avs}} = \frac{\frac{0.5 * V_S^2 * R_{in}}{|Z_S + Z_{in}|^2}}{0.5 * \frac{V_S^2 * R_S}{(R_S + R_S)^2}} = \frac{4 R_{in} R_S}{|Z_S + Z_{in}|^2}, \quad \eta_{MN} = \frac{P_L}{P_{avs}},$
- $\boldsymbol{\eta_{total} = \frac{P_L}{P_{avs}} = \eta_s \eta_{MN}}$
- Change the function $f(V_k) = -\ln(\eta_{tot})$
- $P_{out} = e^{-f(V_n)}$

Outline

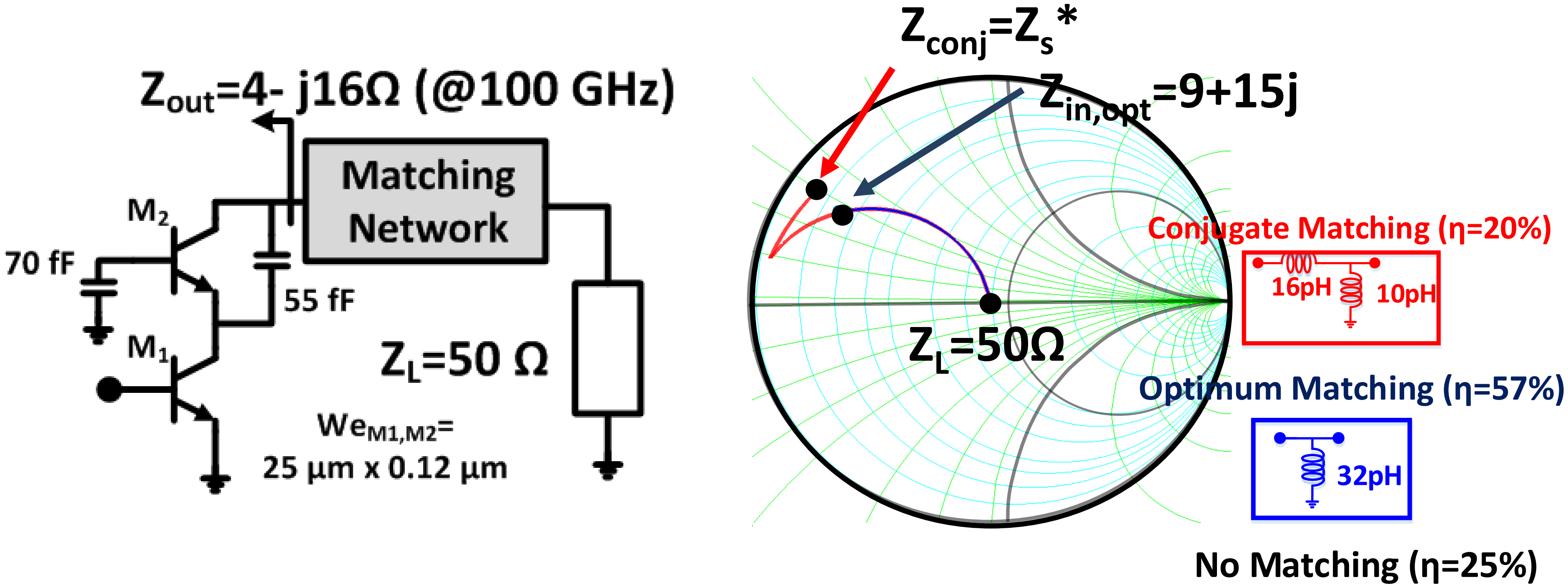
- Concept of Impedance Matching and Motivation
- Non Optimality of Conjugate Matching Networks
- Globally Optimal Impedance Matching Networks
- Design Examples

Example1: Approximation by finite network @100G



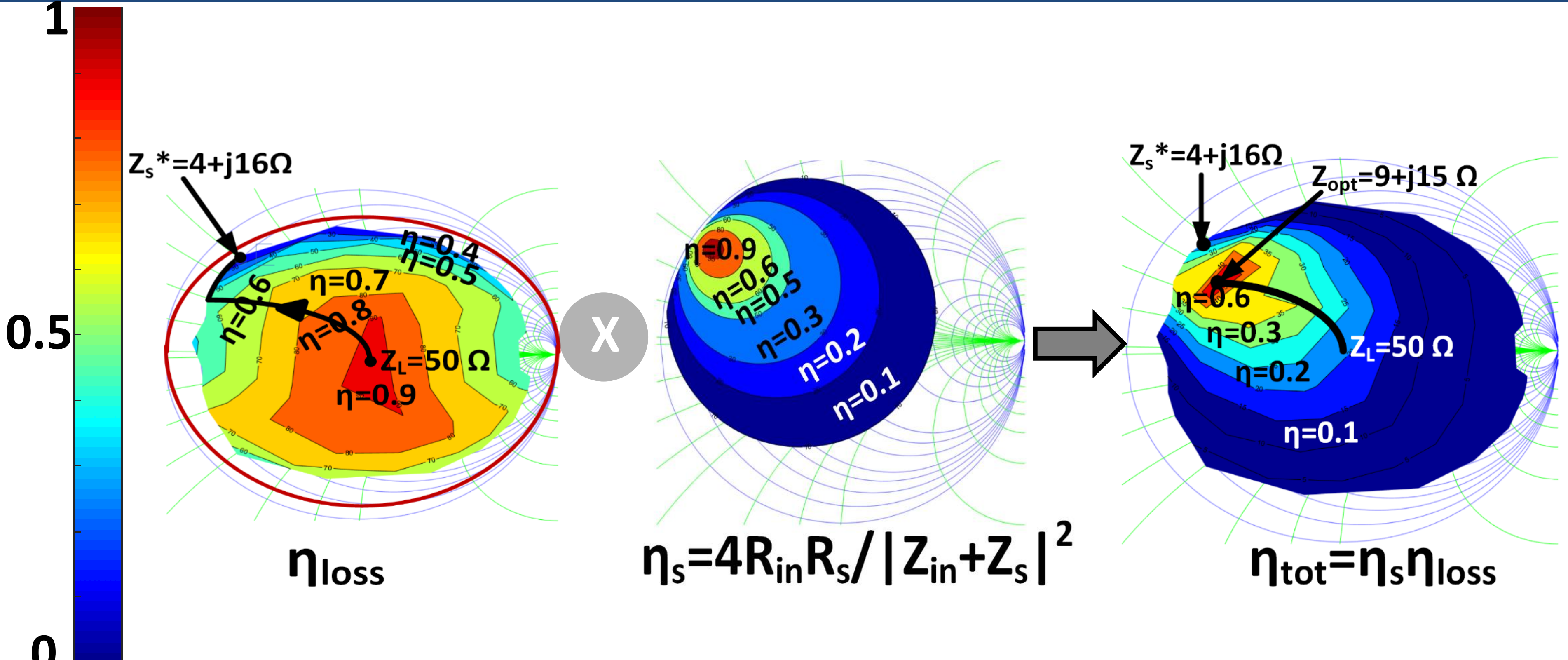
- Globally optimal network provides insight into an optimally efficient network of finite order

Ex. 2: Optimal Matching Vs Optimal Conjugate



- Optimally matching (even no matching) can have significantly higher efficiency than conjugate matching.

Ex. 2: Optimal Matching Vs Optimal Conjugate

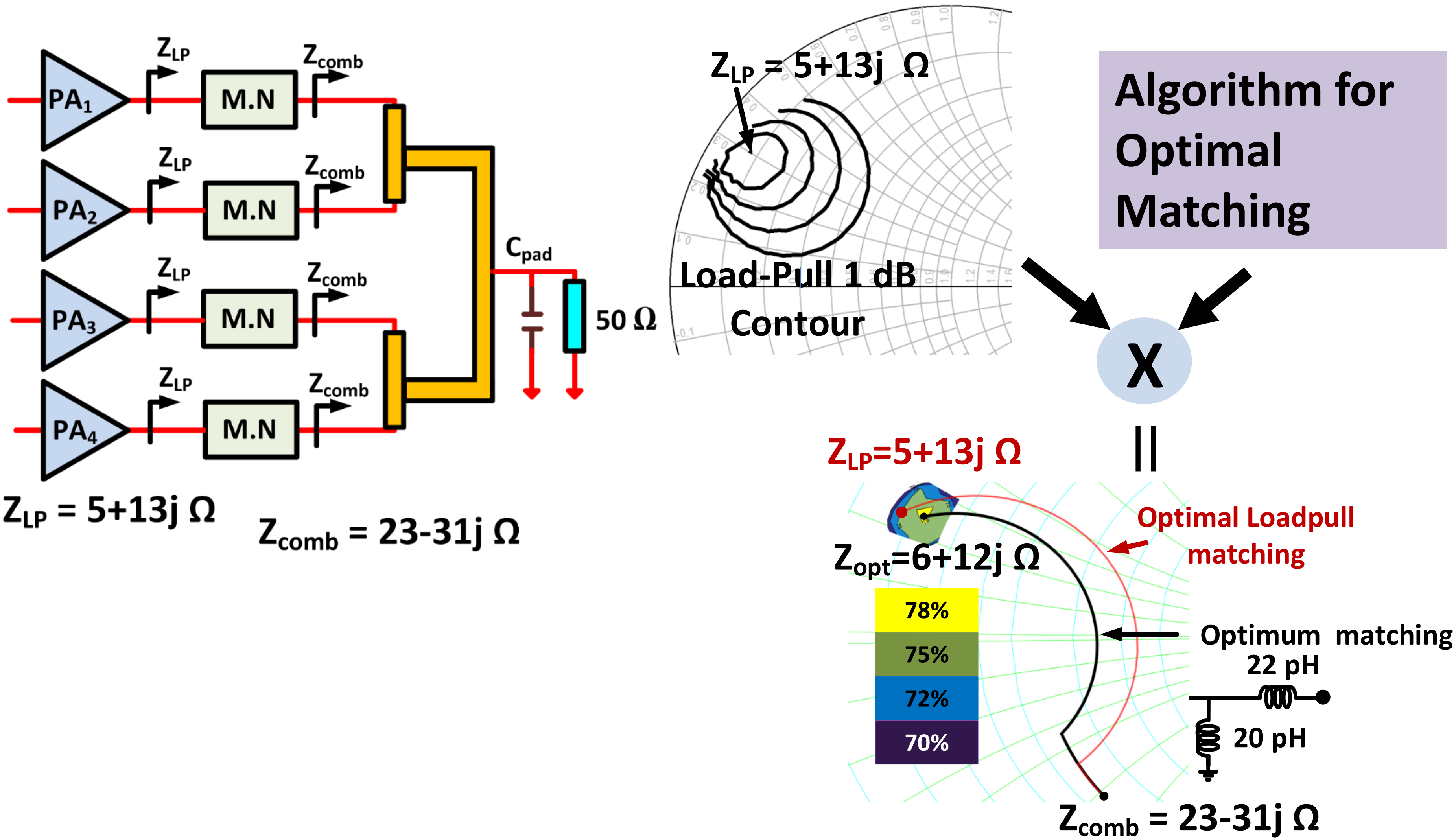


η of MN from 50 Ω to various impedances

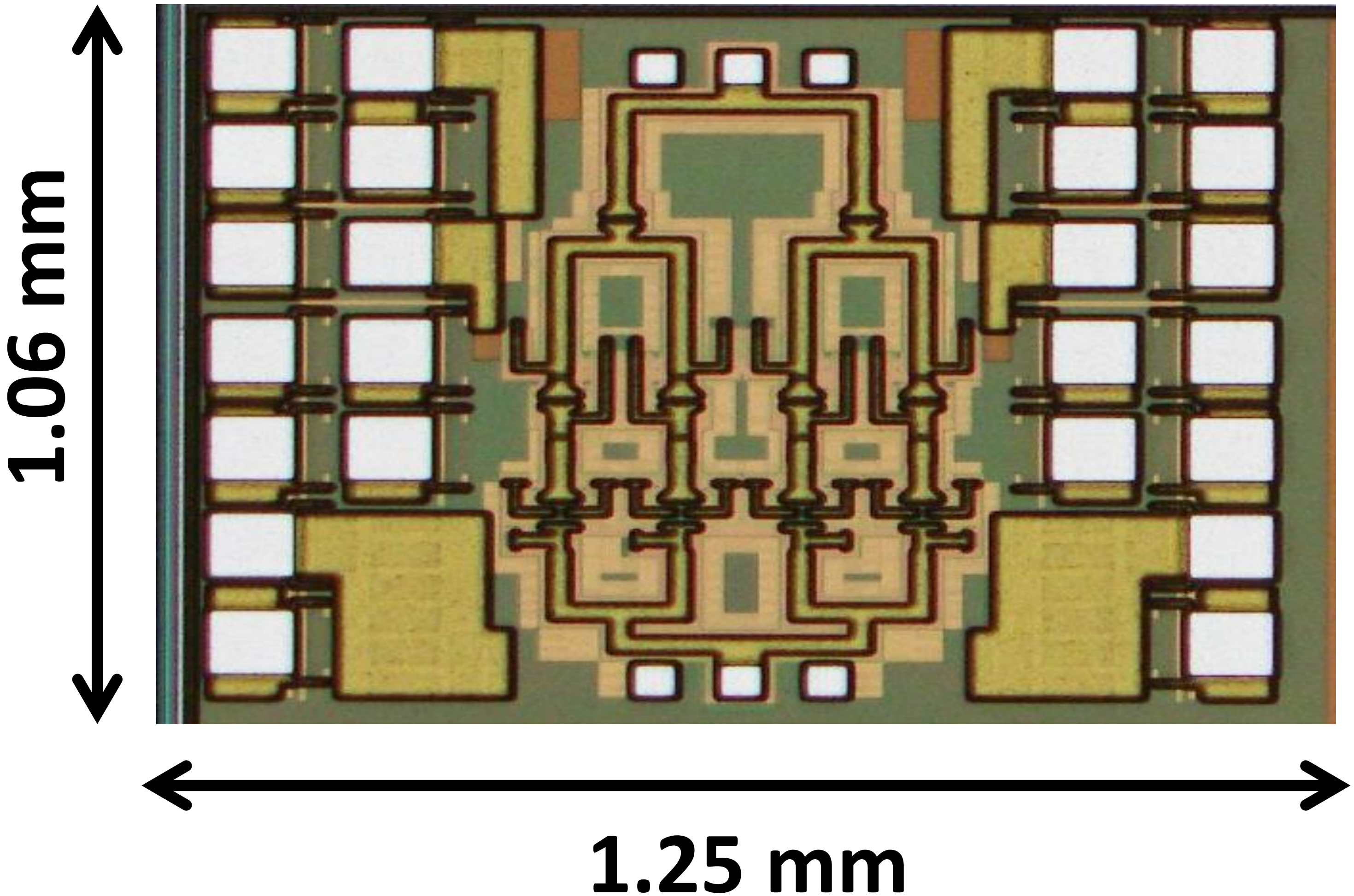
η due to mismatch at the source

Resultant η

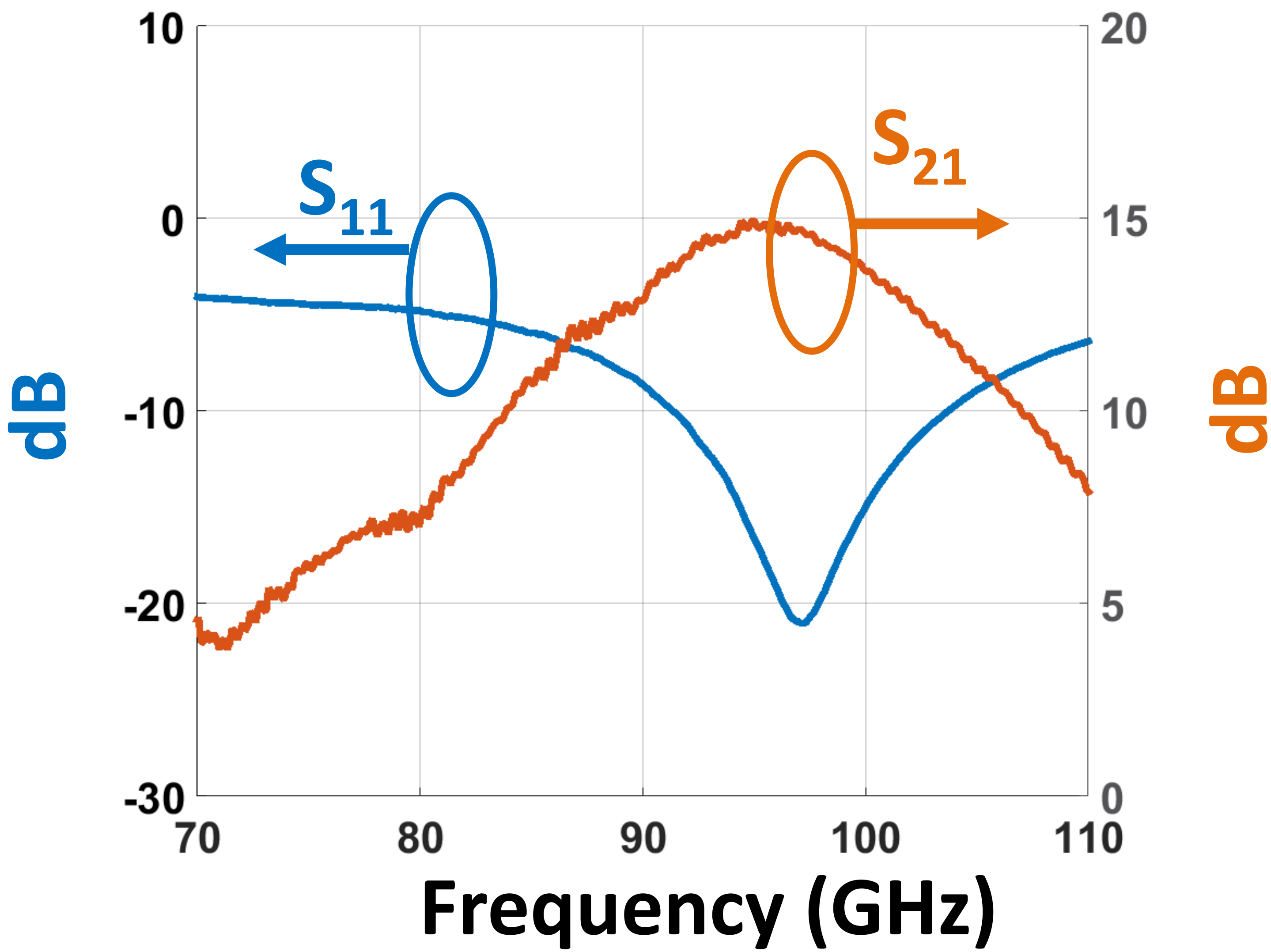
Combiner Network for mm-Wave PA (100 GHz)



100 GHz PA Chip Micrograph



Measured Sparameters of 100 GHz PA chip

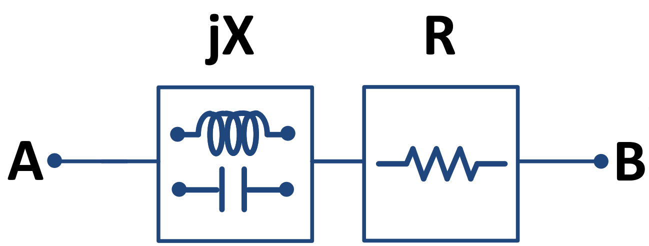


Acknowledgement

- All the members of IMRL, Princeton for technical discussions.
- National Science Foundation and Office of Naval Research.
- MOSIS and IBM for chip fabrication.

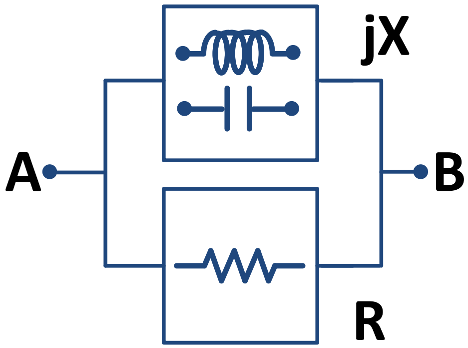
THANK YOU…

Passive Modelling



Series Model

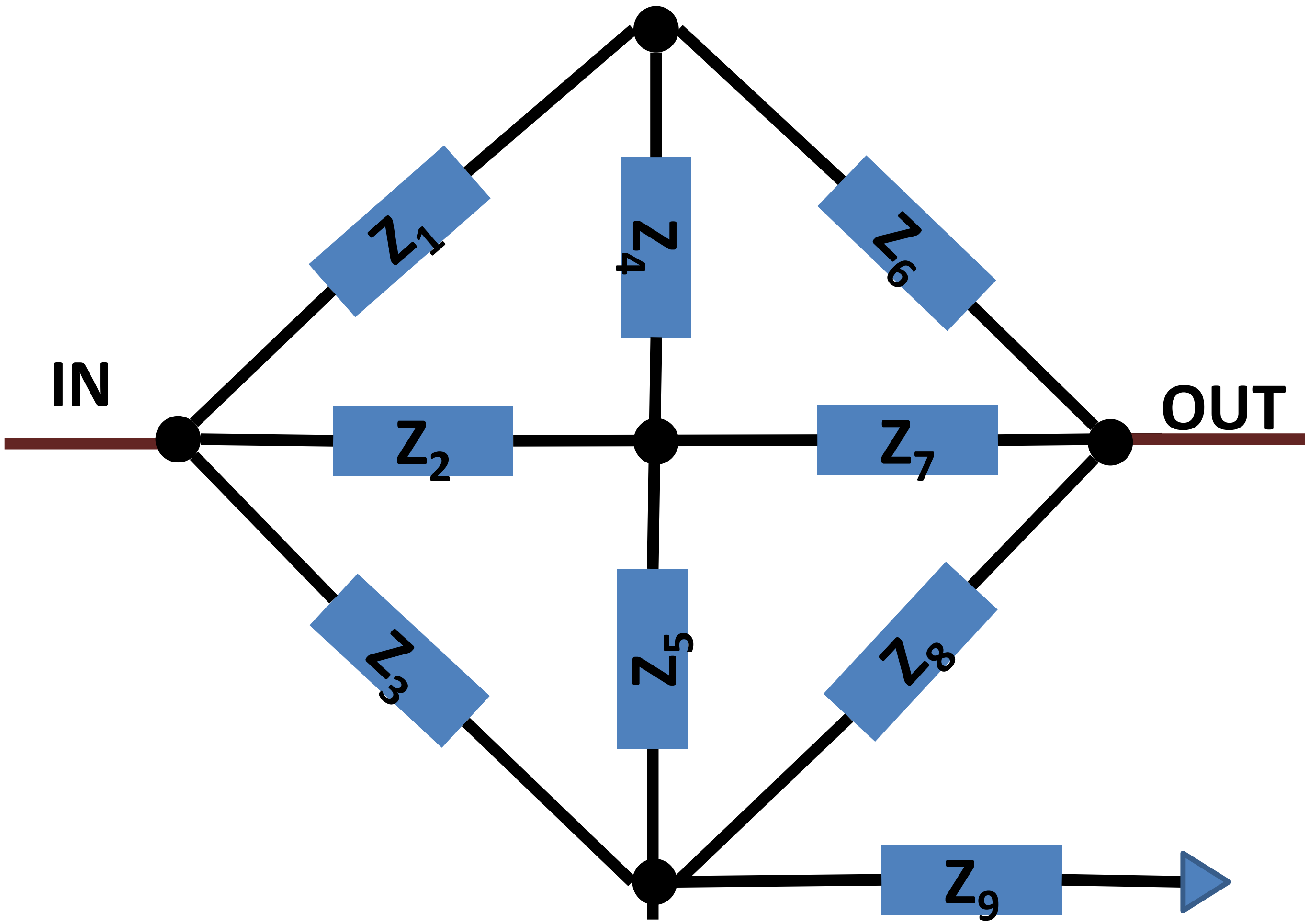
Z_{AB}=R+jX
Y_{AB}=1/Z_{AB} = a+ jb
Q_S=|X/R|=|a/b|



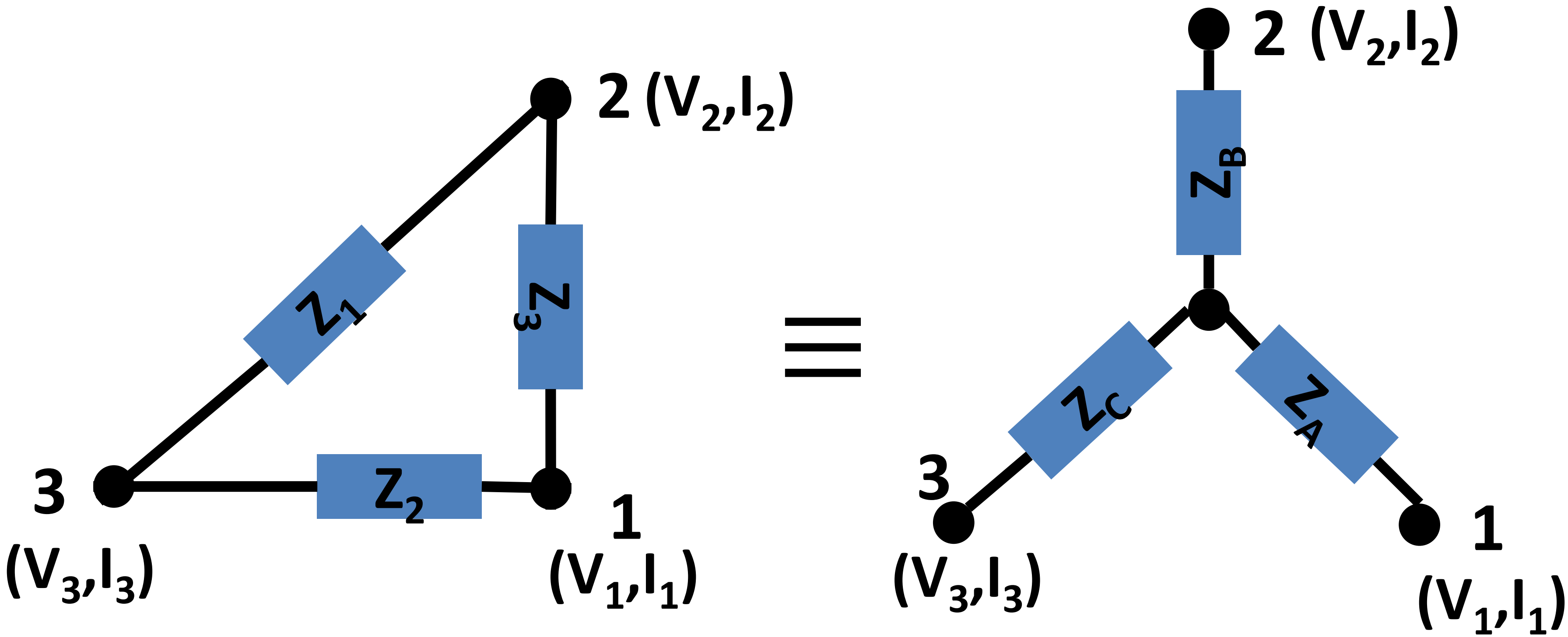
Parallel Model

Y_{AB}=1/R + 1/jX =a+jb
Z_{AB}=1/Y_{AB}
Q_P=|R/X|=|b/a|

Complex circuit → Combination of Series / Parallel Passives

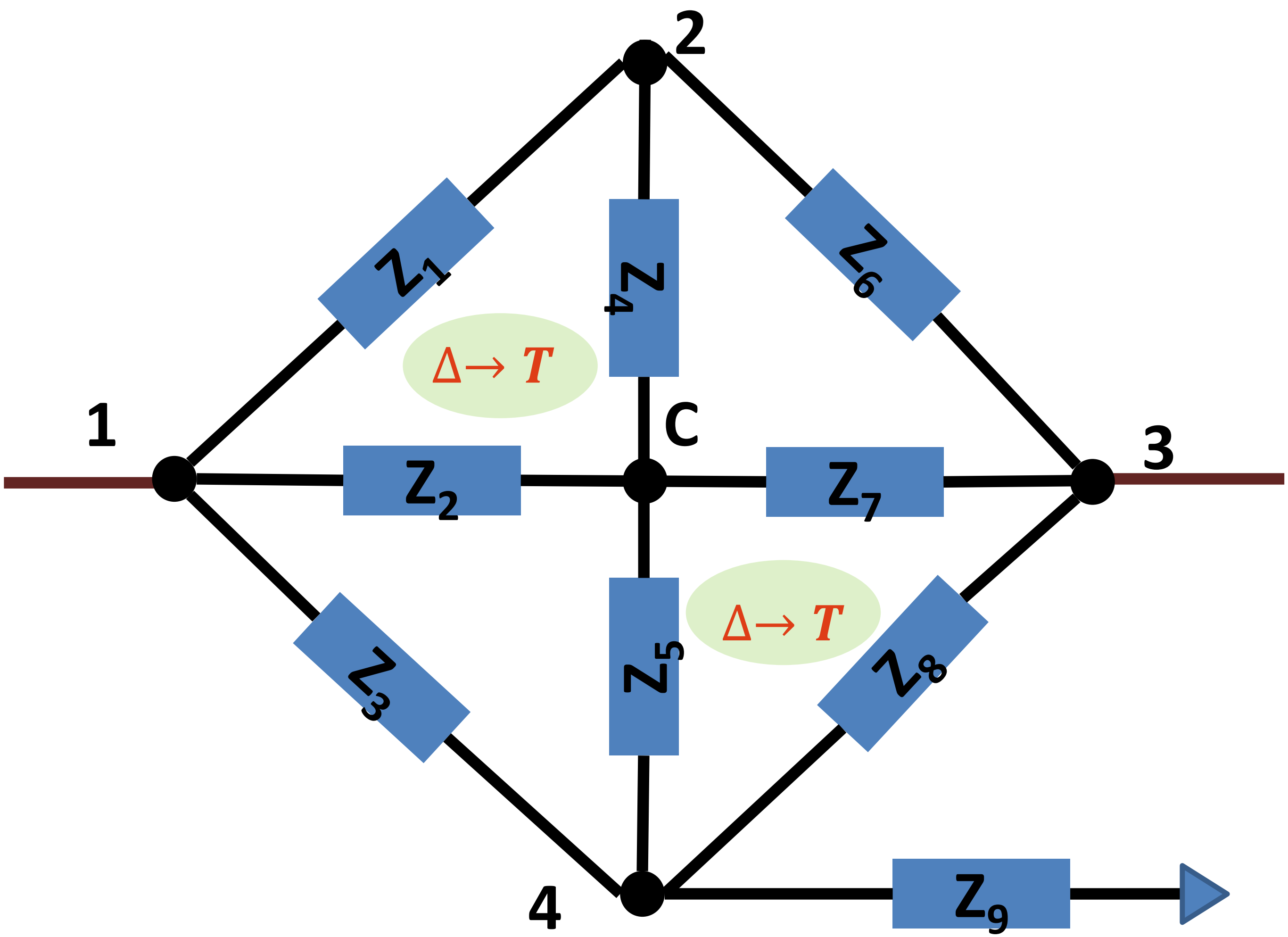


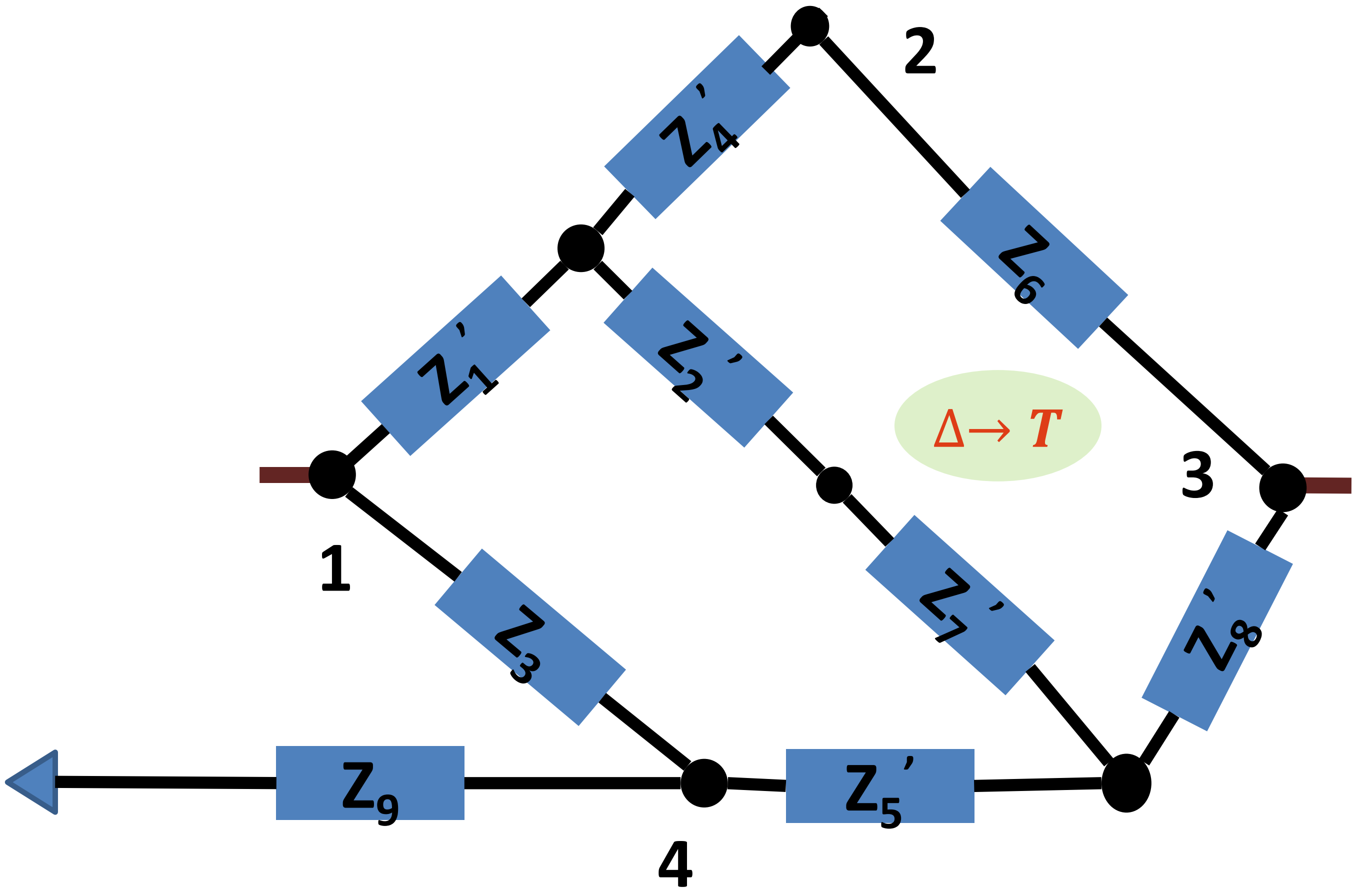
Delta to T conversion



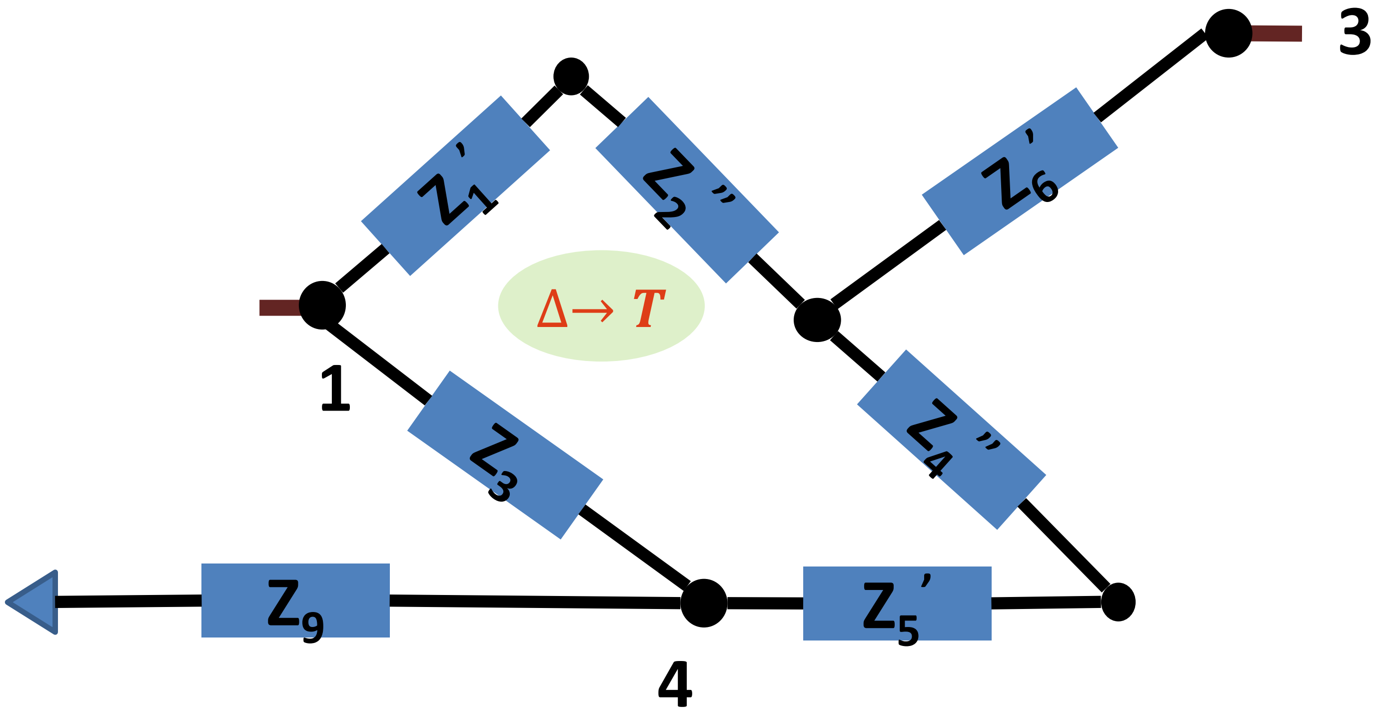
$$Z_A = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} \qquad Z_B = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} \qquad Z_C = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

Complex Circuit

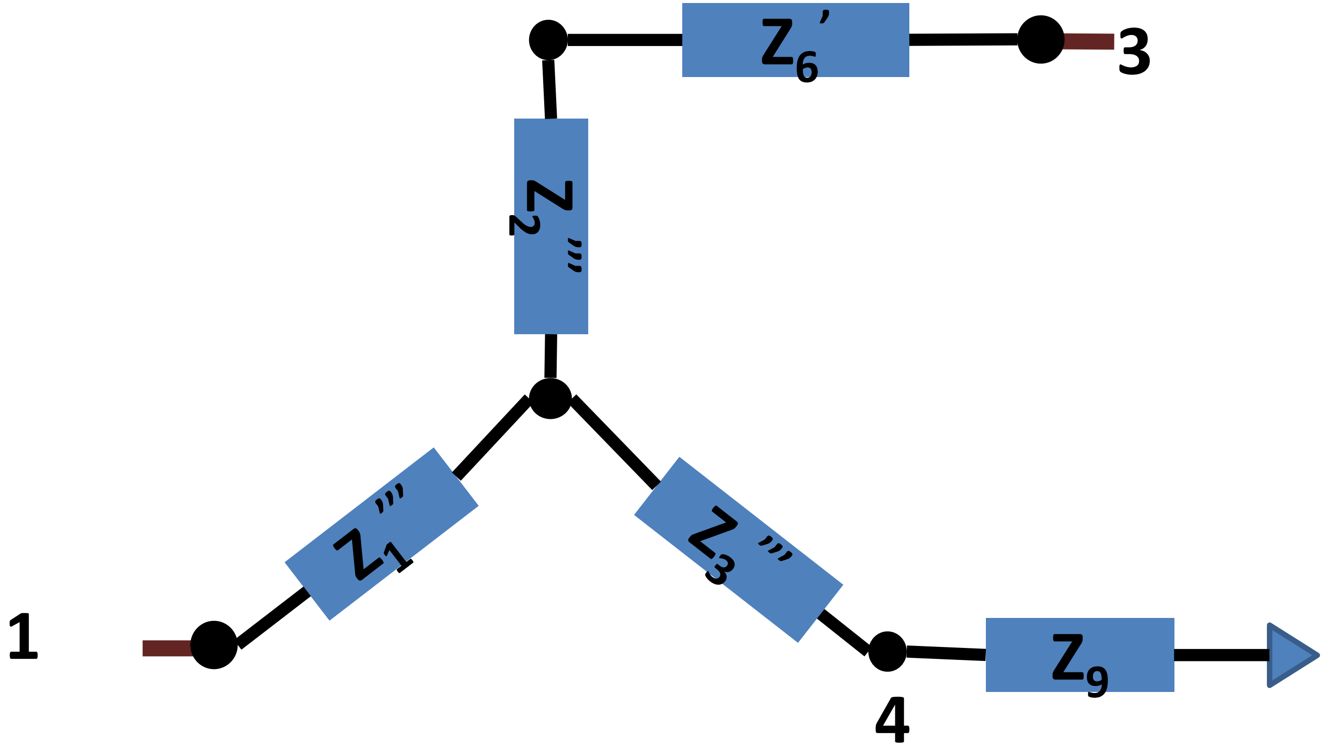




2
Z'4
Z6
Z'1
Z'2
Δ→ T
3
1
Z3
Z'7
Z'8
Z9
Z'5
4

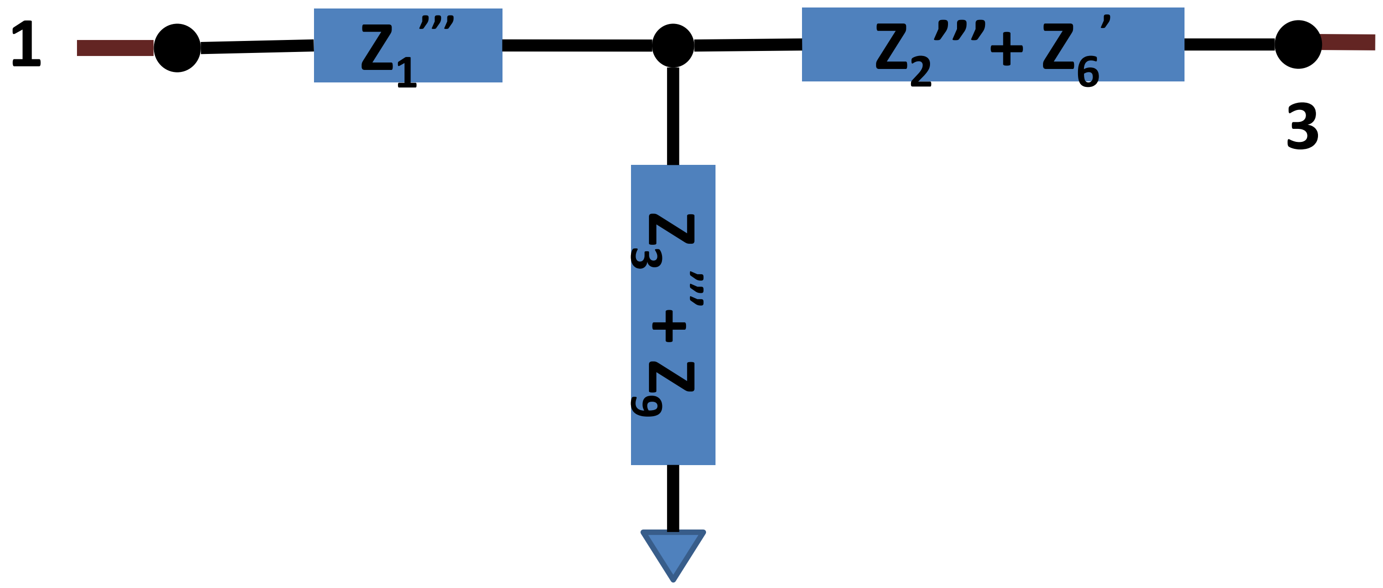


Z1'
Z2''
Z6'
3
Δ→ T
1
Z3
Z4''
Z9
Z5'
4



1
3
4
Z6'
Z2'''
Z1'''
Z3'''
Z9

Reduced form of Complex Circuit



- Every complex combination of passives can be reduced to a combination of series / parallel passives