Fast Statistical Analysis of Rare Circuit Failure Events via Bayesian Scaled-Sigma Sampling for High-Dimensional Variation Space

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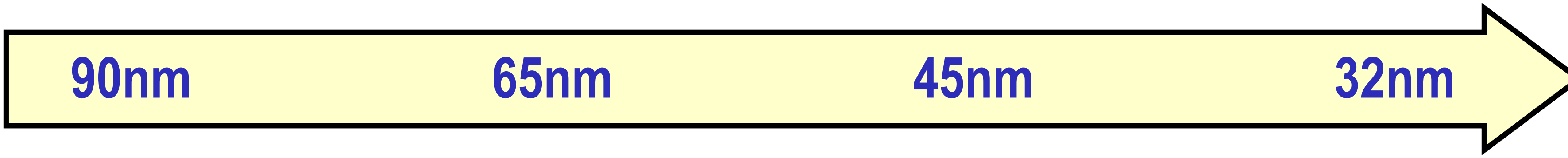
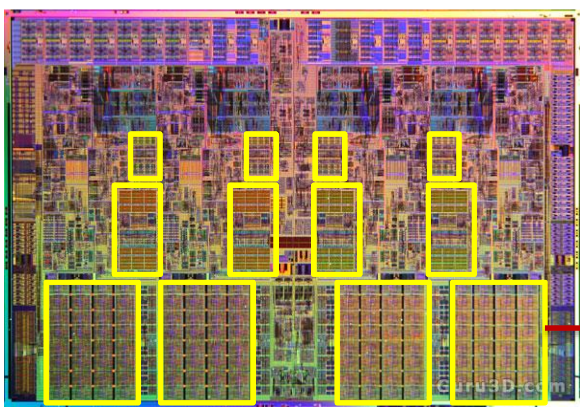
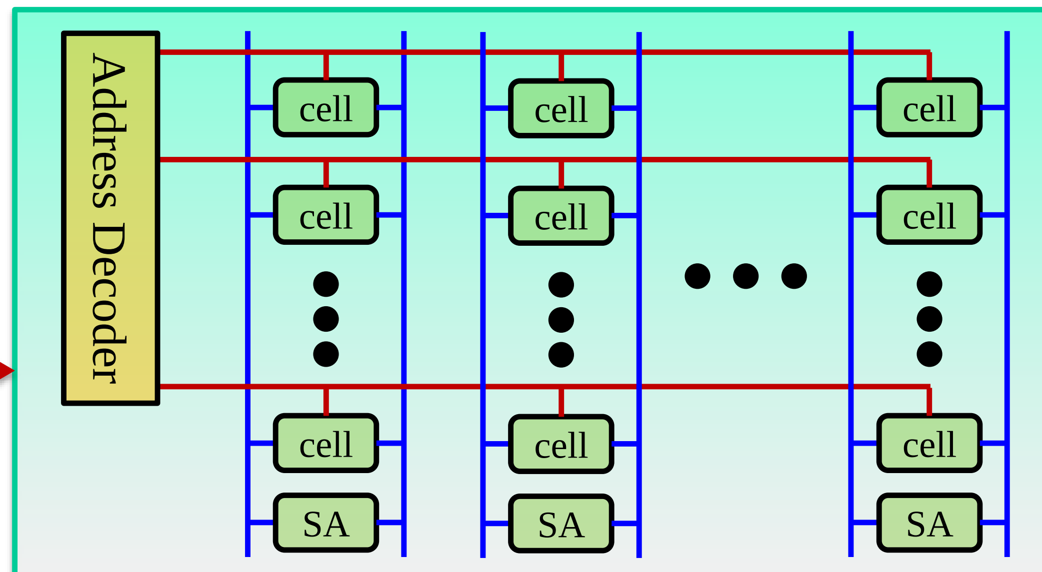
Motivation



- More and more replicated circuit components are integrated on the chip



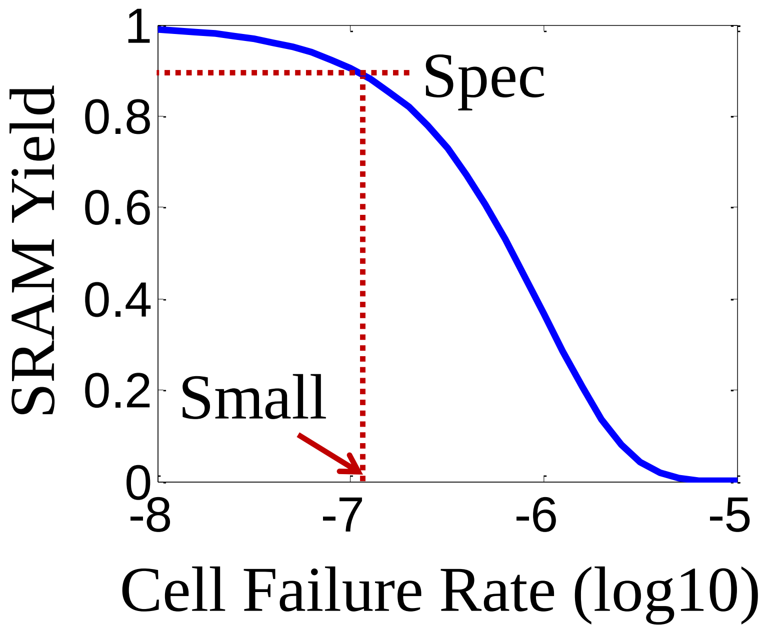
45nm Intel® Core™ i7 Processor



Simplified SRAM Architecture

Motivation

- **Yield requirement**: the failure rate of the replicated circuit component must be extremely small



Assumptions:
1. 1 million cells
2. No ECC
3. No redundancy
4. Only cells fail

- **Time to market**: **fast** statistical tools are highly desired to accurately analyze the **rare** failure event

Traditional Techniques for Low-D Variation Space

- **Importance sampling** (e.g., MIS, MNIS)
- **Statistical blockade** (SB)
- **Integration based techniques** (e.g., NMC, M-C)

[MIS] R. Kanj, R. Joshi and S. Nassif, "Mixture importance sampling and its application to the analysis of SRAM designs in the presence of rare failure events," in *DAC*, pp. 69-72, 2006.
[MNIS] M. Qazi, M. Tikekar, L. Dolecek, D. Shah and A. Chandrakasan, "Loop flattening & spherical sampling: highly efficient model reduction techniques for SRAM yield analysis," in *DATE*, pp. 801-806, 2008.
[SB] A. Singhee and R. Rutenbar, "Statistical blockade: very fast statistical simulation and modeling of rare circuit events, and its application to memory design," in *IEEE TCAD*, vol. 28, no. 8, pp. 1176-1189, Aug. 2009.
[NMC] C. Gu and J. Roychowdhury, "An efficient, fully nonlinear, variability aware non-Monte-Carlo yield estimation procedure with applications to SRAM cells and ring oscillators," in *ASP-DAC*, pp. 754-761, 2008.
[M-C] R. Kanj, R. Joshi, Z. Li, J. Hayes and S. Nassif, "Yield estimation via multi-cones," in *DAC*, pp. 1107-1112, 2012.

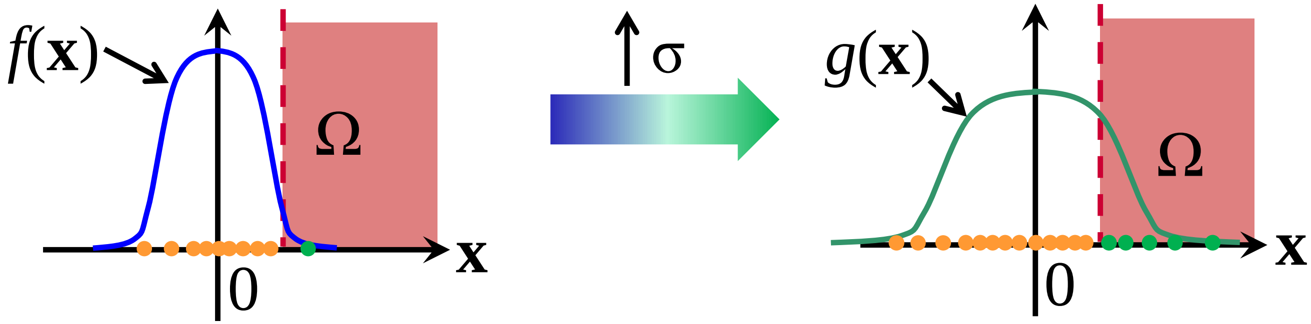
Traditional Techniques for High-D Variation Space

- Rare failure event analysis in a high-dimensional space becomes more and more important [SSS]-[SUS]
  - Dynamic SRAM bit cell stability related to peripherals
  - Rare failure event analysis for other circuits, e.g., SA, DFF
- **Scaled-sigma sampling** (**SSS**): can analyze both continuous and discrete performances of interest
  - E.g., DFF delay, SA output
- **Subset simulation** (**SUS**): can only analyze continuous performances of interest
  - E.g., DFF delay

[SSS] S. Sun, X. Li, H. Liu, K. Luo and B. Gu, "Fast statistical analysis of rare circuit failure events via scaled-sigma sampling for high-dimensional variation space," in *ICCAD*, pp. 478-485, 2013.
[SUS] S. Sun and X. Li, "Fast statistical analysis of rare circuit failure events via subset simulation in high-dimensional variation space," in *ICCAD*, pp. 324-331, 2014.

Traditional Scaled-Sigma Sampling – Overview

- **Idea:** increase the standard deviation (i.e., σ) of $f(\mathbf{x})$ to easily reach the failure region (i.e., Ω)



$$\sigma_g = s \cdot \sigma_f$$

$f(\mathbf{x})$: original PDF

$g(\mathbf{x})$: PDF after scaling σ

P_f : failure rate by sampling $f(\mathbf{x})$

P_g : failure rate by sampling $g(\mathbf{x})$

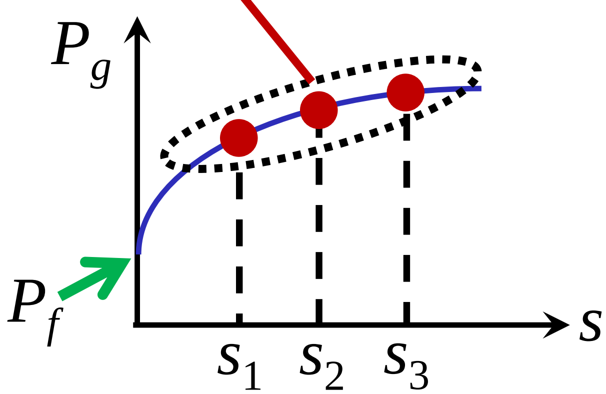
[SSS] S. Sun, X. Li, H. Liu, K. Luo and B. Gu, "Fast statistical analysis of rare circuit failure events via scaled-sigma sampling for high-dimensional variation space," in *ICCAD*, pp. 478-485, 2013.

Traditional Scaled-Sigma Sampling – Overview

- **Idea:** choose several scaling factors, estimate their scaled failure rates P_g, and fit the curve

$$\boldsymbol{\theta} \; : [\alpha \;\; \beta \;\; \gamma]^T$$

$$\mathbf{D} \; : \text{simulation data}$$



$$\log\left(P_g\right) \approx \alpha + \beta \cdot \log\left(s\right) + \frac{\gamma}{s^2}$$

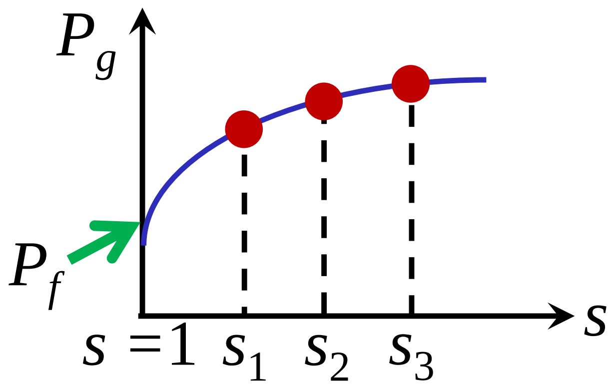
Maximum-likelihood estimation:

$$\max_{\boldsymbol{\theta}} \; pdf\left(\mathbf{D} \middle| \boldsymbol{\theta}\right)$$

Failure rate estimation:

$$P_f = \exp\left(\alpha + \gamma\right)$$

Traditional Scaled-Sigma Sampling – Overview

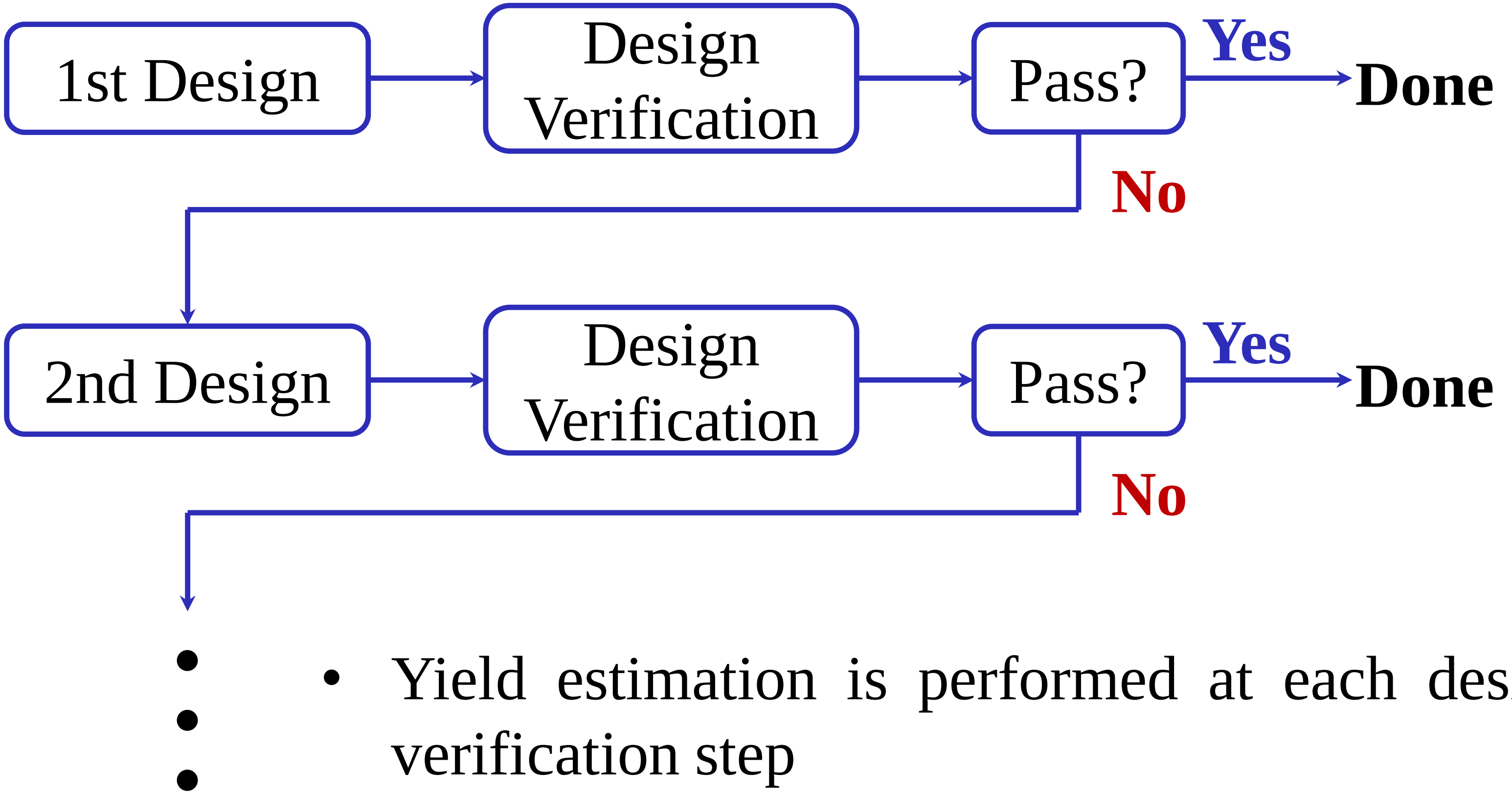


$$\log(P_g) \approx \alpha + \beta \cdot \log(s) + \frac{\gamma}{s^2}$$

- α and β strongly depend on the dimensionality of $\mathbf{x}$, but weakly depend on $\mathbf{\Omega}$
- γ strongly depends on $\mathbf{\Omega}$

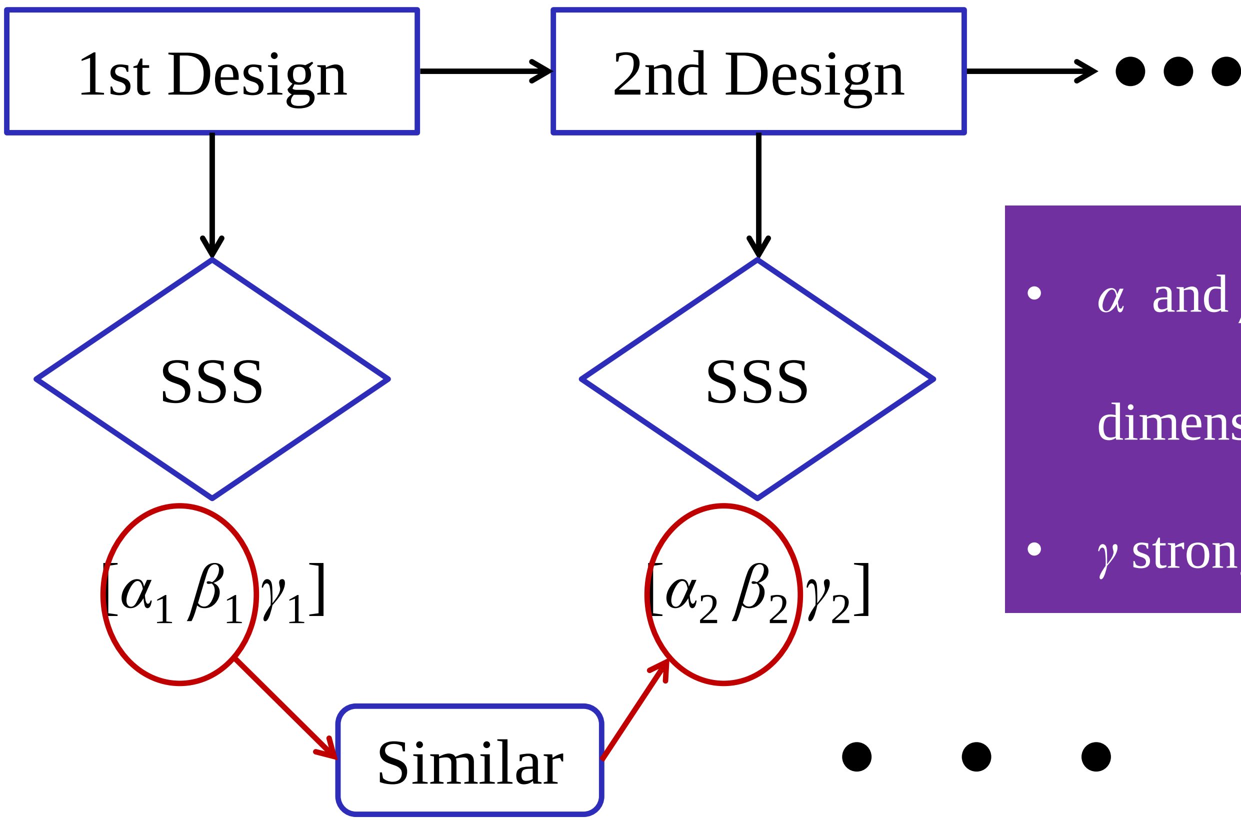
[BSSS] S. Sun and X. Li, "Fast statistical analysis of rare circuit failure events via Bayesian scaled-sigma sampling for high-dimensional variation space," in *CICC*, 2015.

Background



- Yield estimation is performed at each design verification step

- Assume that SSS is applied in yield estimation



- α and β strongly depend on the dimensionality of $\mathbf{x}$
- γ strongly depends on Ω

- **BSSS** is proposed to utilize the "similarity" between different SSS models fitted at different design stages

Outline

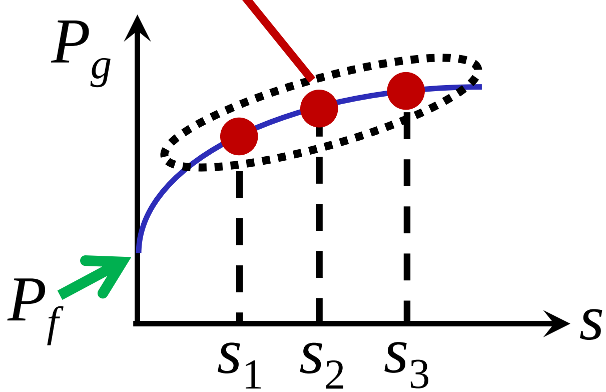
- Motivation
- Background
- **Bayesian Scaled-Sigma Sampling**
- Case Study
- Conclusions

Bayesian Scaled–Sigma Sampling (BSSS)

- **BSSS** formulates a maximum-a-posteriori (MAP) estimation

$\boldsymbol{\theta}$: $[\alpha \;\; \beta \;\; \gamma]^T$

$\mathbf{D}$: simulation data



SSS

$$\max_{\boldsymbol{\theta}} \underbrace{pdf\left(\mathbf{D}\middle|\boldsymbol{\theta}\right)}_{\text{likelihood}}$$

MLE

BSSS

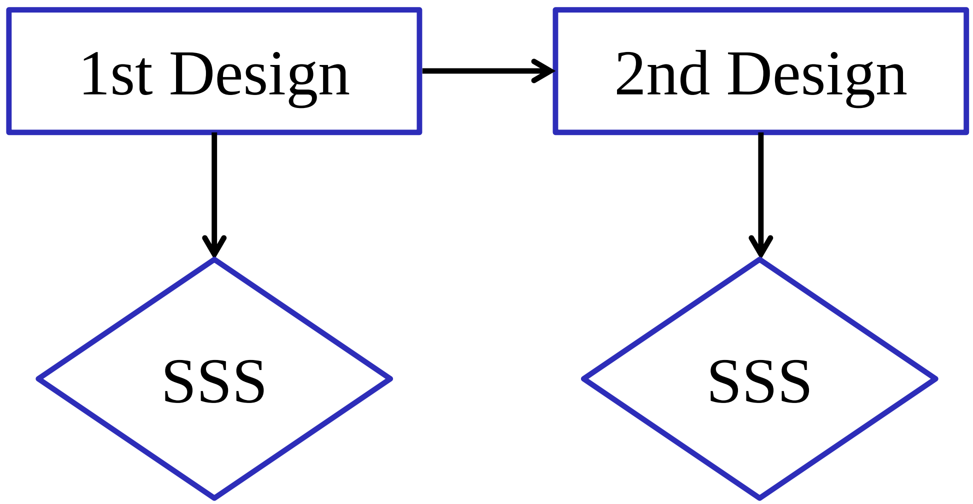
$$\max_{\boldsymbol{\theta}} pdf\left(\boldsymbol{\theta}\middle|\mathbf{D}\right) \propto \underbrace{pdf\left(\mathbf{D}\middle|\boldsymbol{\theta}\right)}_{\text{likelihood}} \cdot \underbrace{pdf\left(\boldsymbol{\theta}\right)}_{\text{prior}}$$

MAP

[MAP] X. Li, F. Wang, S. Sun and C. Gu, "Bayesian model fusion: a statistical framework for efficient pre-silicon validation and post-silicon tuning of complex analog and mixed-signal circuits," in *ICCAD*, pp. 795-802, 2013.

Bayesian Scaled-Sigma Sampling (BSSS)

- **How to define prior?**
  - Assume that the SSS model for the 1st design is known, and we are learning the SSS model for the 2nd design



$\boldsymbol{\theta}_1 = [\alpha_1\ \beta_1\ \gamma_1]$ $\boldsymbol{\theta}_2 = [\alpha_2\ \beta_2\ \gamma_2]$

Bayesian Scaled–Sigma Sampling (BSSS)

How to define prior?

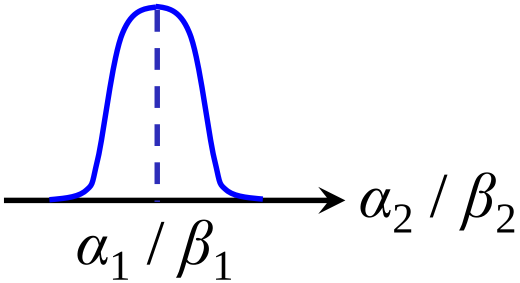
- Assume that three model coefficients are independent. Correlation information will be further learned by MAP estimation

$$pdf\left(\boldsymbol{\theta}_2\right) = pdf\left(\alpha_2\right) \cdot pdf\left(\beta_2\right) \cdot pdf\left(\gamma_2\right)$$

- Define α_2 and β_2 as Normal random variables

$$\alpha_2 \sim N\left(\alpha_1, \sigma_\alpha^2\right)$$

$$\beta_2 \sim N\left(\beta_1, \sigma_\beta^2\right)$$



- Solve σ_α and σ_β by MLE

$$\max_{\sigma_\alpha, \sigma_\beta} pdf\left(\mathbf{D} \middle| \sigma_\alpha, \sigma_\beta\right)$$

Bayesian Scaled-Sigma Sampling (BSSS)

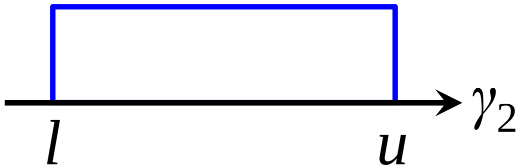
How to define prior?

- Assume that three model coefficients are independent. Correlation information will be further learned by MAP estimation

$$pdf\left(\boldsymbol{\theta}_2\right) = pdf\left(\alpha_2\right) \cdot pdf\left(\beta_2\right) \cdot pdf\left(\gamma_2\right)$$

- Define γ_2 as a uniform random variable (non-informative prior)

$$\gamma_2 \sim U\left(l, u\right)$$



- l is set to a very small number, u is set to a very large number

Bayesian Scaled–Sigma Sampling (BSSS)

Algorithm Flow

1. Given α_1 and β_1 from the previous design, l, and u
2. Collect simulation data $\mathbf{D}$
3. Solve σ_α and σ_β by MLE

$$\max_{\sigma_\alpha,\sigma_\beta} pdf\left(\mathbf{D}\middle|\sigma_\alpha,\sigma_\beta\right)$$

4. Form the prior distribution

$$pdf\left(\boldsymbol{\theta}_2\right) = pdf\left(\alpha_2\right)\cdot pdf\left(\beta_2\right)\cdot pdf\left(\gamma_2\right)$$

$$\alpha_2 \sim N\left(\alpha_1,\sigma_\alpha^2\right) \qquad \beta_2 \sim N\left(\beta_1,\sigma_\beta^2\right) \qquad \gamma_2 \sim U\left(l,u\right)$$

5. Estimate $\boldsymbol{\theta}_2 = [\alpha_2\ \beta_2\ \gamma_2]$

$$\max_{\boldsymbol{\theta}_2} pdf\left(\mathbf{D}\middle|\boldsymbol{\theta}_2\right)\cdot pdf\left(\boldsymbol{\theta}_2\right)$$

6. Calculate the failure rate P_f

$$P_f = \exp\left(\alpha_2 + \gamma_2\right)$$

Outline

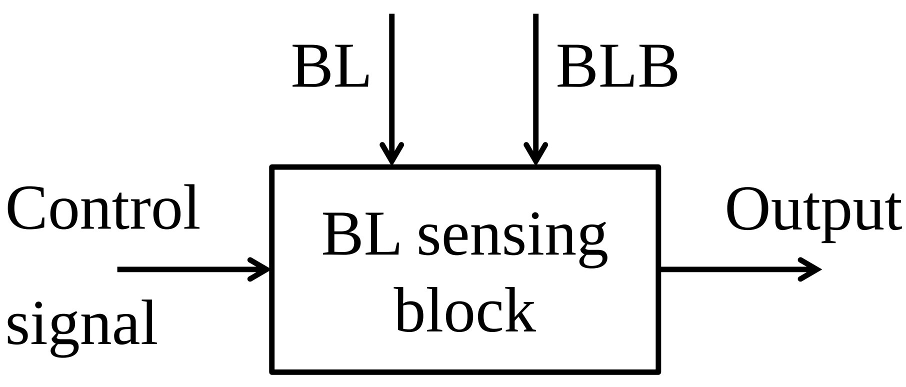
- Motivation
- Background
- Bayesian Scaled-Sigma Sampling
- Case Study
- Conclusions

Case Study

- Designed in 45nm CMOS process

- Each transistor is composed of several multipliers
  - 4 independent variables are used to model process variations for each multiplier

- Initially, BL is set to 1.1V, and BLB is 1.2V. If the output of SA is 0, SA is considered as “PASS”. Otherwise, “FAIL”



Sense amplifier (SA) block

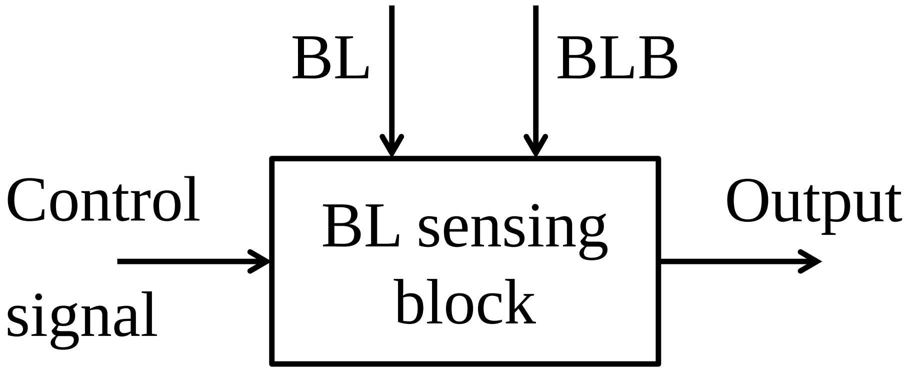
Case Study

- Four different methods are implemented:
  1. MC (brute-force Monte Carlo)
     - Provide the golden failure rate
  2. MNIS (traditional importance sampling)
     - 2000 simulations are used to find the importance distribution
  3. SSS (traditional scaled-sigma sampling)
     - 5 scaling factors are empirically chosen
  4. BSSS (proposed)
     - Use the same simulation data as SSS

[MNIS] M. Qazi, M. Tikekar, L. Dolecek, D. Shah and A. Chandrakasan, "Loop flattening & spherical sampling: highly efficient model reduction techniques for SRAM yield analysis," in *DATE*, pp. 801-806, 2008.
[SSS] S. Sun, X. Li, H. Liu, K. Luo and B. Gu, "Fast statistical analysis of rare circuit failure events via scaled-sigma sampling for high-dimensional variation space," in *ICCAD*, pp. 478-485, 2013.

Case Study

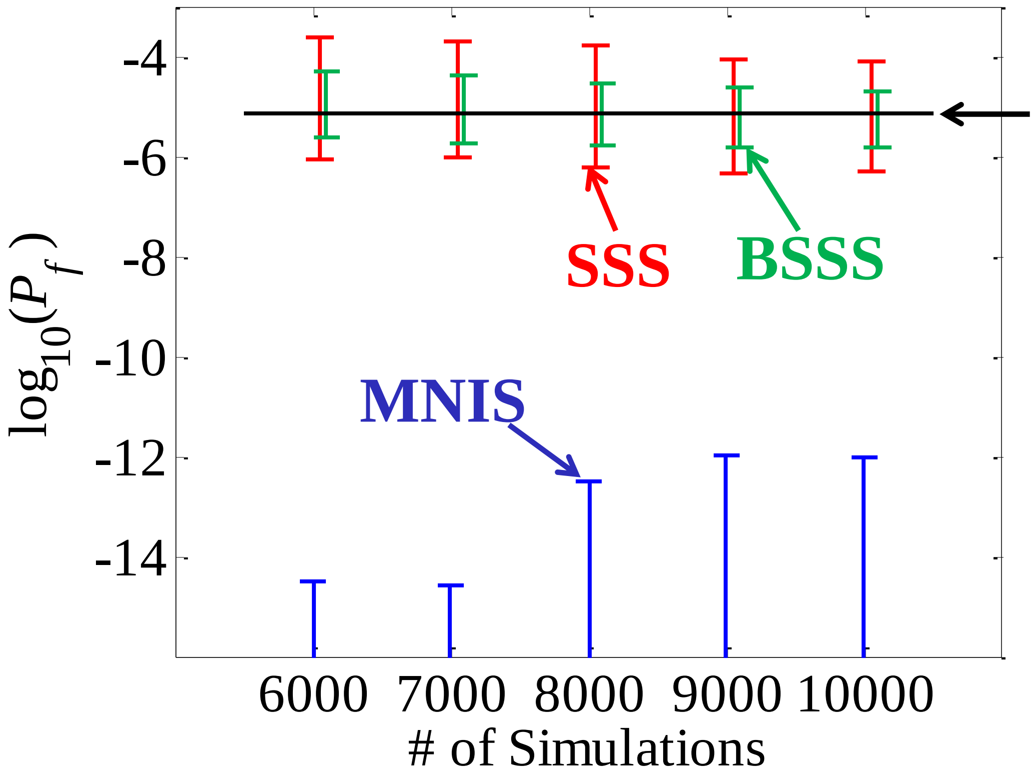
- 536 independent random variables for the 1st SA design
- The estimated failure rate by SSS with 10^4 samples is 7.3×10^{-5}
- We need to tune the 1st SA design to improve its performance



Sense amplifier (SA) block

Case Study

- A 2nd SA design is tuned from the 1st design
  - 552 independent random variables for the 2nd SA design
- The “golden” failure rate estimated by MC with 3.5×10^{6} samples is 7.1×10^{-6}



Golden failure rate

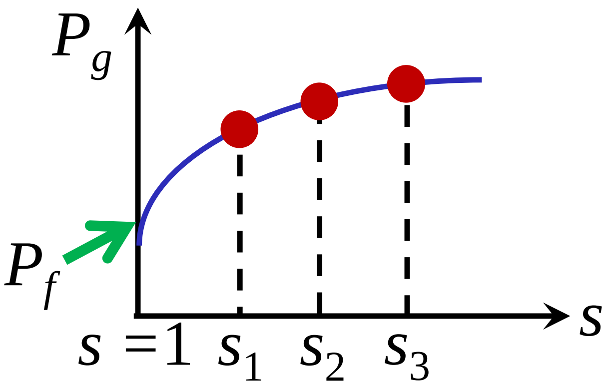
95% confidence intervals are compared

Conclusions

- A novel BSSS method is proposed to accurately estimate the rare failure rates for nanoscale ICs in a high-D space
- BSSS extends SSS by using the “similarity” between different SSS models fitted at different design stages
- SA is used to demonstrate the proposed BSSS method
  - The dimensionality of the variation space is more than a few hundred
  - BSSS achieves superior estimation accuracy over traditional MNIS and SSS

Thank You!

Traditional Scaled-Sigma Sampling – Overview



Ω : failure region

M : # of parameters

$\mathbf{x}^{(k)}$: a hyper-rectangular in Ω

$\Delta\mathbf{x}$: the volume of a hyper-rectangular

T : # of dominant hyper-rectangular

$$\log\left(P_g\right) \approx \alpha + \beta \cdot \log\left(s\right) + \frac{\gamma}{s^2}$$

$$\alpha = \log\left[\frac{\Delta\mathbf{x}}{\left(2\pi\right)^{M/2}}\right] + \log(T)$$

$$\beta = -M$$

May not change a lot after changing Ω

$$\gamma = -\min_{k\in\Omega}\left(\left\|\mathbf{x}^{(k)}\right\|_2^2\right)\Big/2$$

Can change a lot after changing Ω

Case Study

- The golden failure rate of the 2nd design is 7.1×10^{-6} which is estimated by MC with 3.5×10^{6} random samples

Table 1. Failure rate and 95% CI $[P_f^L, P_f^U]$. Golden P_f is 7.1×10^{-6}

# of Sims		6000	7000	8000	9000	10000
MNIS	P_f^L	0	0	0	0	0
	P_f	1.5×10^{-15}	1.2×10^{-15}	1.1×10^{-13}	4.1×10^{-13}	3.6×10^{-13}
	P_f^U	3.1×10^{-15}	2.5×10^{-15}	3.1×10^{-13}	1.1×10^{-12}	9.2×10^{-13}
SSS	P_f^L	9.0×10^{-7}	9.4×10^{-7}	6.2×10^{-7}	4.5×10^{-7}	5.1×10^{-7}
	P_f	4.4×10^{-5}	3.8×10^{-5}	2.0×10^{-5}	1.1×10^{-5}	1.0×10^{-5}
	P_f^U	2.3×10^{-4}	2.0×10^{-4}	1.7×10^{-4}	8.8×10^{-5}	7.9×10^{-5}
BSSS	P_f^L	2.4×10^{-6}	1.8×10^{-6}	1.7×10^{-6}	1.5×10^{-6}	1.5×10^{-6}
	P_f	1.3×10^{-5}	9.0×10^{-6}	8.0×10^{-6}	7.0×10^{-6}	6.2×10^{-6}
	P_f^U	4.8×10^{-5}	4.1×10^{-5}	2.8×10^{-5}	2.4×10^{-5}	2.0×10^{-5}