Methods for Finding Globally Maximum-efficiency Impedance Matching Networks with Lossy Passives

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***Abstract*—Impedance transformation using on-chip passive elements is ubiquitously used in RF and mm-Wave circuits and systems for optimal power matching, interstage and noise matching, and high-efficiency power delivery to the antenna by power amplifiers. While conjugate matching gives optimal efficiency for lossless passives, the results are markedly different when constituent passives have finite quality factors. Given the load and source impedances, there may be infinite ways to achieve the transformation, albeit each incurring different loss. In this paper, we investigate the methods to deduce the global maximum efficiency of power transfer between two arbitrary impedances with lossy passives. This paper also proposes methods to combine this with nonlinear load-pull simulations for optimal efficiency combiner and matching network for integrated PAs. To the best of the authors' knowledge, this is the first comprehensive analysis of globally optimal impedance transformation networks between arbitrary impedances with lossy passives.**

I. Introduction

Impedance matching attempts to achieve conjugate matching for maximum power transfer. However, losses in the network can be significant, especially at high frequencies and as such, it is not obvious what is the best impedance to match to and what is the minimal loss path. Traditionally, multi-dimensional optimization has been used to synthesize impedance transformation networks with minimum loss [1]. However such a procedure is often heuristic and ad-hoc, the number of elements in the network is often a guess estimate. In this paper, we investigate the fundamental limits of power transfer between two arbitrary impedances using lossy matching passives and the globally optimal impedance transformation network.

II. Non optimality of Conjugate Impedance Matching Networks

Let us consider the situation shown in Fig. 1. We are interested in maximum power transfer to a load R_L from a source with impedance $Z_s = R_s(1 - jQ_s)$ through a two port matching network consisting of passive elements. This is a typical case which we encounter in small-signal amplifiers. However this argument can be extended to load-pull matching of power amplifiers. In the lossless case, the passive network should transform the impedance R_L to the conjugate of source impedance, i.e., $Z_{in} = Z_s^* = R_s(1 + jQ_s)$. Maximizing the power input looking from the source therefore maximizes the power transfer to the load. However if the network elements are lossy, conjugate impedance transformation (if possible) does not guarantee maximum power transfer to R_L since a large fraction may be lost in the passive matching network. Therefore at the frequency of interest, we are interested in determining the lowest loss network , global maximum power transfer possible to R_L, the global optimum input impedance $Z_{in,opt}$ and if such a transformation can be achieved using lossy elements. We will try to motivate this with two examples, as shown below.

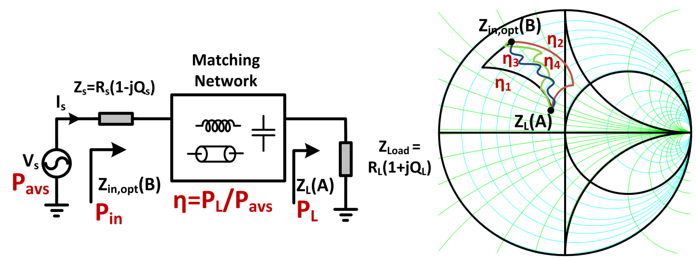

Fig. 1. Optimally efficient impedance transformation network to maximize power transfer efficiency $\eta = \frac{P_L}{P_{AVS}}$ with lossy passives. *Graphically, this is equivalent to finding the lowest loss path from Z_L (A) to $Z_{in,opt}$ (B) among the infinite possible paths and determining the optimal value of $Z_{in,opt}$.* In general $Z_{in,opt} \neq Z_s^*$.

Example 1: Consider a case, where a common-emitter power amplifier delivers 250 mW at 100 GHz. This requires an impedance transformation from R_L= 50 Ω to $Z_s^* = 2 + 2j\Omega$. In this example, we transform the impedance to the exact desired impedance of $Z_{in} = 2+2j\Omega$. Fig. 2 shows six possible matching networks (six possible paths on the Smith Chart), all of which have markedly different efficiencies due to the finite quality factors of the passives ($Q_{cap} = 10$, $Q_{ind} = 10$ in this example). The efficiency of this network critically affects the overall efficiency of the transmitter and it is neither intuitive nor obvious what is the most optimal path on the Smith Chart.

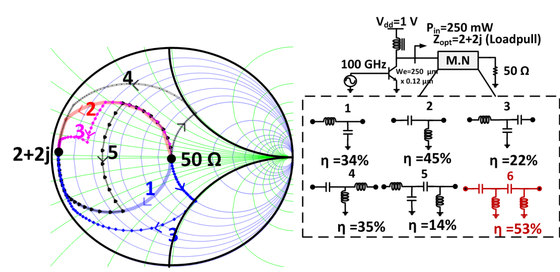

Fig. 2. Example 1: Six possible matching networks with different efficiencies and the corresponding paths on Smith Chart for R_L= 50 Ω to $Z_s^* = 2+2j\Omega$ at 100 GHz. The passives have finite quality factors ($Q_{cap} = 10$, $Q_{ind} = 10$).

Example 2: Let's consider another example to demonstrate that conjugate matching is, in general, sub-optimal for lossy networks. Let the source impedance $Z_s = R_s(1 - jQ_s)$ (capacitive, $Q_s > 0$), $Z_L = R_L(1+jQ_L)$ (inductive, $Q_L > 0$) and $Z_{in} = R_{in} + jX_{in}$. The total input power to the network is given by $P_{in} = I_s^2 R_{in}/2$ and the total input reactive power $P_{reac,in} = I_s^2 X_{in}/2 = P_{ind} - P_{cap} + P_{reac,L}$, where P_{ind}, P_{cap} are the reactive power stored in inductors and capacitors of the matching network respectively, and $P_{reac,L}$ is the reactive power in the load given by $P_{reac,L} = Q_L P_L$[1]

Optimal Network: The loss in the matching network P_{loss} is related to the reactive powers as $P_{loss} = \frac{P_{ind}}{Q_{ind}} + \frac{P_{cap}}{Q_{cap}} \geq |\frac{P_{ind}-P_{cap}}{Q_{ind}}| = \frac{P_{reac,in}-P_{reac,L}}{Q_{ind}}$*.Therefore, the loss in the matching network is minimized when $P_{cap} = 0$ i.e., matching network strictly composed of inductors with no capacitors which allows us to reach the theoretically lowest loss [2].*

Optimal Z_{in}: As argued before, $Z_{in,opt} \neq Z_s^*$ in general. In fact, $Z_{in,opt}$ can be analytically derived as follows. For $P_{cap} = 0$, the power delivered to the load $P_L = P_{in} - P_{loss} = \frac{1}{2} I_s^2 (R_{in} - \frac{X_{in}}{Q_{ind}}) + \frac{P_{reac,L}}{Q_{ind}} = \frac{\frac{1}{2}\frac{V_s^2(R_{in}-\frac{X_{in}}{Q_{ind}})}{(R_{in}+R_s)^2+(X_{in}-R_sQ_s)^2}}{1-Q_L/Q_{ind}}$. At $Z_{in,opt} = R_{in,opt} + jX_{in,opt}$, $\frac{\partial P_L}{\partial R_{in}} = 0$ and $\frac{\partial P_L}{X_{in}} = 0$. This gives

$$R_{in,opt} = R_s \frac{2Q_{ind}Q_s + Q_{ind}^2 - 1}{1 + Q_{ind}^2}$$
$$X_{in,opt} = R_s \frac{Q_{ind}^2 Q_s - Q_s - 2Q_{ind}}{1 + Q_{ind}^2} \quad (1)$$

The globally maximal efficiency of power transfer, in such a case, is given by

$$\eta_{max} = \frac{P_{L,max}}{P_{avs}} = \frac{1 + Q_{ind}^2}{Q_{ind}(Q_{ind} + Q_s)(1 - Q_L/Q_{ind})} \quad (2)$$

It can be seen that as $Q_L \to \infty$, $\eta \to 1$ as expected while as $Q_s \to \infty$, $\eta \to 0$ which also makes intuitive sense [2].

Note1: While the analysis presented provides globally optimal solution, given Z_L, there is only a limited range in Smith Chart which can be reached with a purely inductive network with no capacitors. This is shown in Fig. 3. Additionally, there are constraints on the nature of source impedance which can be matched to $Z_{in,opt}$ with purely inductive networks. As an example, from (2), $Q_{ind} > Q_L$, and $Q_s(Q_{ind} - 1/Q_{ind}) > 2$ (from (1)). Fig. 3 shows an example with $Z_s = 10 - 20j\Omega$ and $Z_{load} = 30\Omega$ ($Q_s = 2, Q_L = 0$) and $Q_{ind} = 5$ which meet the conditions. In such a case, we obtain $Z_{in,opt} = 17 + 15j\Omega \neq Z_s^*$ (1), with $P_{L,max} = 0.74P_{avs}$ and $P_{in} = 0.9P_{avs}$. This implies that the optimally efficient network only transfers 90% of P_{avs} into the network and 74% of P_{avs} reaches Z_L. *This is a globally optimal solution for the given quality factor of passives and it is not possible to reach higher efficiency with any combination of the passive elements.*

[1] Reactive power at frequency ω in an inductor L and C with current i is defined as $P_{ind} = \omega W_m = \frac{i^2\omega L}{2}$ and $P_{cap} = \omega W_e = \frac{i^2}{2\omega C}$, where W_m and W_e are the magnetic and electrical energy stored in the inductor and capacitor.

Note2: As is evident from the derivations of ((1)-(2)), optimal efficiency is achieved as long as the network is composed of only inductors, independent of the topology. Therefore, if $Z_{in,opt}$ lies in the 'reachable' space in the Smith Chart as shown in Fig. 3, all possible networks/paths consisting of only inductors between Z_L and $Z_{in,opt}$ have identical efficiency (74% in this example) and represent the globally optimal solution.[2] The efficiency contours are plotted in Fig. 3.

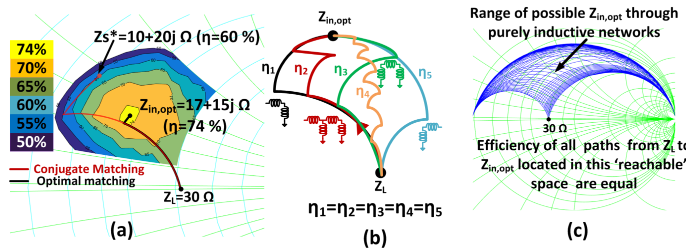

Fig. 3. (a) For lossy networks, $Z_{in,opt} \neq Z_s^*$. Efficiency contours and $Z_{in,opt}$ are shown for $Z_s = 10 - 20j\Omega$ and $Z_{load} = 30\Omega$ and $Q_{ind} = 5$. (b) All possible networks/paths consisting of only inductors between Z_L and $Z_{in,opt}$ have identical efficiency (c) Possible range of impedances which can be reached from Z_L (= 30 Ω in this example) using only inductors. If $Z_{in,opt}$ lies in this region, (1)-(2) represent the globally optimal solution.

III. Algorithm for Globally Maximum-Efficiency Matching Network for any Z_L and Z_s

In this section, we will consider the problem shown in Fig. 1 which attempts to find the globally optimal-efficiency impedance transformation network between two arbitrary impedances Z_L and Z_s. We will address two parts to this problem. Firstly, we will address the lowest loss path between two impedances Z_L and $Z_{in,opt}$. Secondly, we will also determine $Z_{in,opt}$ for given impedances Z_s and Z_L and the passive quality factors Q_L and Q_c. The method to arrive at the optimal path is described below.

A. Optimal Conjugate Matching Networks(OCM)

1) *Any arbitrary path in the Smith Chart can be approximated with a combination of parallel and series passive elements as shown in Fig. 4. Therefore, first discretize the impedance and admittance space of the Smith Chart and locate the load impedance (Z_L).*
2) *Efficiency of power transfer in a path of length Δp_i between impedances Z_i and $Z_i + \Delta Z_i$ realized with a series element ΔX and ΔY with quality factor Q can be evaluated as shown in Fig. 4*

$$\eta_i = \begin{cases} \frac{1}{1+\frac{|\Delta X|}{QR}} & \textit{if } i_{th} \textit{ element is added in series,} \\ \frac{1}{1+\frac{|\Delta S|}{QG}} & \textit{if } i_{th} \textit{ element is added in parallel} \end{cases}$$

3) *Total efficiency in any path 'p' is, therefore, given by $\eta_{tot,p} = \prod_{i=1}^{N} \eta_i$, where N = total number of elements.*

[2] If the reactive energy is purely capacitive, (i.e., Z_s is inductive) and the matching network is strictly composed of capacitors, the similar observation can be noticed.

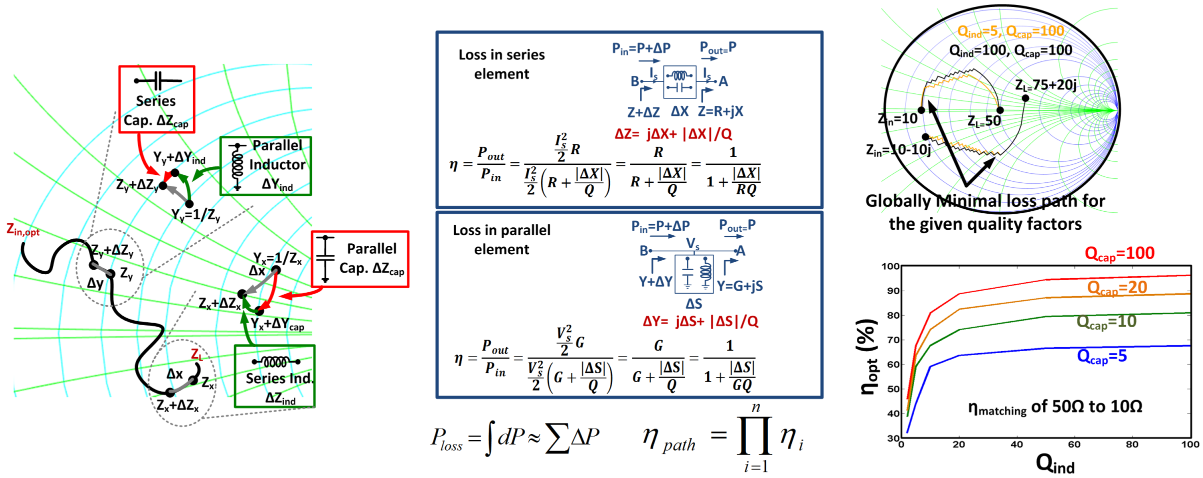

Fig. 4. Algorithm for optimal path between two impedances Z_L and $Z_{in,opt}$. Any arbitrary path can be approximated with a set of parallel and series passive elements and the efficiency of power transfer of any path can be evaluated as shown. Designating each path loss as the cost function or distance, finding the globally optimal path is reduced to finding the path between two nodes in graph with the minimum distance. The figure also shows the examples of optimal efficiency paths and globally optimal efficiency achievable with lossy passives of various quality factors.

4) *In this way, the entire Smith chart can be discretized into small paths Δp_i with a 'distance' or cost function of Δp_i being defined as $\Delta d_i = -\ln(\eta_i)$. Therefore, total distance or total cost for any path 'p' between Z_L and $Z_{in,opt}$ can be evaluated as $d_{tot,p} = \sum_{i=1}^{N} \Delta d_i$.*
5) *The problem of globally optimal path between the nodes Z_L and $Z_{in,opt}$ now reduces to finding the path with the shortest'distance' d_{tot}.*
6) *There are various algorithms that can accomplish this. As an example, Dijkstra's algorithm [3] can be applied to find the 'shortest' distance from load impedance to source impedance with complexity order $O(|V^2|)$, where V is the number of nodes in the graph. Dijkstra's algorithm applied to this problem is briefly described below*
   a) *Mark all points/nodes in the discretized Smith Chart as unvisited. Set the current node as the node of load impedance (Z_L) and set its nodelength (which represents the shortest distance of current node from Z_L) as zero.*
   b) *Mark the current node as visited and add it to the visited group. If all the nodes are visited then stop.*
   c) *Find the non−visited node (V_k) at a shortest distance from the visited group.*
   d) *Store the nodelength of this node (V_k) as $d_{min}(V_k)$ and define function $f(V_k) = |Z(V_k) - Z_s^*|$. Also store its immediate previous node.*
   e) *Go to step (b)*
7) *Find the node (V_n) that minimizes $f(V_n)$ and back trace the path. Max Power transfer efficiency of the matching network is given by $\eta_{opt,conj} = e^{-d_{min}(V_k)}$.*

As an example, Fig. 4 shows the globally optimal efficiency path between two sets of impedances $Z_L = 75 + 20j\Omega$, $Z_{in} = 10 - 10j\Omega$ and $Z_L = 50\Omega, Z_{in} = 10\Omega$ for two sets of quality factors and a Smith Chart discretized to 2000 points. The figure also shows the global maximum efficiency achievable with impedance matching between $Z_L = 50\Omega$ and $Z_{in} = 10\Omega$ as the quality factor of the passives vary.

B. Optimum Matching (OM)

Generally, for lossy networks as shown before, $Z_{in,opt} \neq Z_s^*$. Additionally, it may also not be possible to reach Z_s^* with the given lossy passives. In order to find the globally optimal power transfer efficiency between Z_s and Z_L, the algorithm can be modified to find $Z_{in,opt}$ and the optimal path to reach the impedance. The power transfer efficiency can be modified to reflect the impedance mismatch in such a case as

$$\eta_{opt} = \frac{P_L}{P_{avs}} = \frac{P_L}{P_{in}}\frac{P_{in}}{P_{avs}} = \eta_{loss}\eta_s \tag{3}$$

where $\eta_s = \frac{4R_{in}R_s}{|Z_{in}+Z_s|^2}$ represents the efficiency of power transfer from the source to the network (Fig. 1). In case of conjugate matching, $\eta_s = 1$. Therefore, in order to find the optimum impedance matching path which delivers maximum power to the load, the function in step 6(d) of the algorithm is changed as $f(V_k) = -\ln(\eta_s) + d_{min}(V_k)$. In this case, the net optimal efficiency is given by $\eta_{opt} = e^{-f(V_n)}$, where V_n is the optimal node.

IV. Design Examples

A. Approximating the optimal path with finite-order network

While the optimal path with the lowest loss is not practically realizable, the algorithm provides us with the starting point to approximate the highest efficiency path with finite order networks. Fig. 5 illustrates this with an example where the $Z_{load} = 100\Omega$ and $Z_s = 1 - 1j$ (@ 100 GHz) with $Q_{ind} = 10, Q_{cap} = 10$. The optimum conjugate matching path and its approximation to one, two and three networks are shown in Fig. 5. It can be seen that the three-stage network with properly chosen passive elements can reach close to the optimal efficiency.

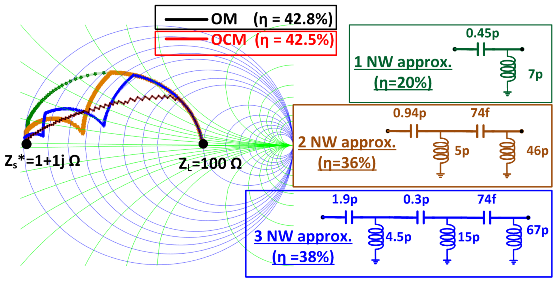

Fig. 5. Examples of approximation of the optimal path with finite-order networks.

B. Optimal Matching Vs Optimal Conjugate Matching

Fig. 6 illustrates the case where there is a significant difference in power transfer efficiencies between optimal matching and optimal conjugate matching. Consider the case where the output impedance of a small signal amplifier is $Z_s = 4 - 16j\Omega$ (@ 100GHz) and the passives have quality factors $Q_{ind} = 5, Q_{cap} = 10$. Based on the algorithm, Fig. 6 shows the contours of η_{loss} and η_s (3) from $Z_L = 50\Omega$. The net power transfer efficiency (η_{tot}) is the product of the two efficiencies. This results in an optimal impedance of $Z_{in,opt} = 9 + 15j\Omega \neq Z_s^*$ where the power transfer efficiency is 57%, while for conjugate matching, maximum possible efficiency is 20%. Infact, the efficiency of power transfer with no matching network is higher at 25%. As is shown in Fig. 6, this situation arises because the the conjugate impedance is located towards the edge of space of possible transformed impedances with the given quality factors, as shown in Fig. 6. Intuitively, the extra network to reach the exact point Z_s^* induces excessive loss degrading efficiency dramatically.

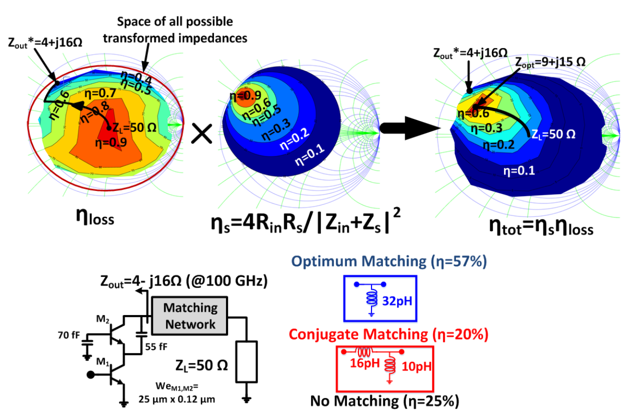

Fig. 6. Optimal Matching Vs Optimal Conjugate Matching. For lossy passives, the set of all possible transformed impedances only cover a subset of the Smith Chart. The loss for brute force conjugate matching can increase dramatically if Z_s^* is located towards the edge of the 'reachable' space.

C. Optimal Matching with Nonlinear Loadpull Simulations

The algorithm for optimal matching network can be merged with nonlinear load-pull simulations for optimal power amplifier combiner and matching network design. This can be incorporated in (3) where η_s measures the efficiency degradation due to impedance mismatch. For a small-signal amplifier, this can be easily expressed as $\eta_s = \frac{4R_{in}R_s}{|Z_{in}+Z_s|^2}$. In case of a power amplifier, the decrease in power delivered from the load-pull can be obtained from the nonlinear load-pull simulations. The core algorithm remains unchanged.

An example of a 4-way power combiner at 100 GHz in 0.13 μm BiCMOS process is shown in Fig. 7. The impedance looking into the combiner by each of the four unit amplifiers is $23 - 31j\Omega$ while the load-pull impedance of each amplifier (W_e=48×0.12μm^2) is given by $Z_{LP} = 5 + 13j\Omega$. The available passive quality factors are $Q_{ind} = 16, Q_{cap} = 10$. Fig. 7 shows the results when the nonlinear loadpull simulations are incorporated into the algorithm to generate the optimal matching network and optimal conjugate matching network. In this case, both efficiencies are similar and the details are shown in Fig. 7.

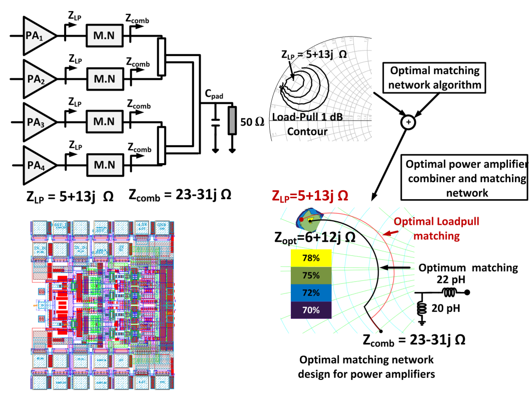

Fig. 7. Optimal matching network method combined with nonlinear load-pull simulations enables globally maximal efficiency matching network and/or combiner for power amplifiers. The figure shows the layout for the 4-stage power amplifier with optimal combiner at 100 GHz.

V. Conclusion

This paper presents methods to analyze and derive globally optimal efficiency impedance matching networks with lossy passives. We derive optimal impedance to be matched for lossy passives, range of possible matching achievable and globally optimum Smith chart trajectory to minimize loss in impedance matching. The algorithm can be easily combined with nonlinear load-pull simulations for optimal power amplifier combiner and matching network design.

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